Household Disaggregation in WiNDC

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Overview

- First version of WiNDC featured a state level dataset with a single representative agent by region.
- Provided means for spatially denominated distributional analysis, but not within consumer types.
- A key advantage of IMPLAN was its disaggregation of regional consumer demands and incomes by household income groups.
- Many ways to go about this type of disaggregation. Incomes vs. expenditures.
- We approach this problem from the income side. Key challenges: denominate reasonable transfer income, understand income tax liabilities, savings, capital ownership vs. demands.
- Unexpected challenge and focus of the modeling application: static vs. steady state calibration.

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Statistics of Income – IRS.

- Provides income data based on administrative income tax return data. Match 1040 form line numbers with state by income group.

Symbol	Household categories
<10k	AGI under $10,000
10_25k	AGI $10,000 under $25,000
25_50k	AGI $25,000 under $50,000
50_75k	AGI $50,000 under $75,000
75_100k	AGI $75,000 under $100,000
100_200k	AGI $100,000 under $200,000
200_500k	AGI $200,000 under $500,000
500k_1m	AGI $500,000 under $1,000,000
>1m	AGI over $1,000,000

- Tax-filing unit vs. household.
- We focus on 2016, but have processed data from 2013-2016. Data availability goes back to 1996.

Mapping income categories in the data

We've aggregated the following data to use for our calibration. Note these can be kept separate if needed.

- Labor income: wages, salaries, and tips.
- Capital/interest income: taxable interest, dividends, business income, capital gains, rental real estate, royalties, trusts, partnerships, and S corporations, pensions and annuities, IRA distributions.
- Transfer income: unemployment compensation, social security benefits, state and local tax refund.
- Savings income: IRA deductions.
- Federal income tax liabilities

Some 1040 data elements were not included in state level SOI data: non-taxable portion of pension and social security benefits, alimony received, farm income.

National income shares in SOI

Symbol	Total Income	Wages	Interest	Transfers
<10k	117.4	0.73	0.26	0.01
10_25k	564.1	0.76	0.23	0.02
25_50k	1304.1	0.81	0.15	0.03
50_75k	1258.8	0.77	0.18	0.05
75_100k	1141.8	0.75	0.20	0.06
100_200k	2558.2	0.75	0.22	0.04
200_500k	1621.4	0.68	0.30	0.02
500k_1m	597.6	0.56	0.44	0.01
>1m	1368.2	0.30	0.70	0.00

Note that regional heterogeneity exists at the state level.

Calibration routine

SOI data cover only the taxable portion of needed income characteristics. We use a two stage matrix balancing routine to "fill in the gaps" while getting as close as possible to SOI data.

- First: pin down disaggregated region by income group income components.
- Second: given these levels, solve for consumer demands using GTAP income elasticities of demand.

Features of the calibration routine:

- Capital earnings and savings markets clear at a single national price. Assume all earnings go through the "New York Stock Exchange". Circumvents issue of capital demand vs. ownership.
- Assume that geography of savings is independent of investment.
- Income taxes fixed to SOI levels. Inflates tax revenues.
- Domestic vs. foreign savings. BEA data on foreign direct investment.

Savings and Transfers

Limited information is available from SOI on transfers and savings – two very important aspects of welfare. Heuristics needed to pin down "reasonable" levels.

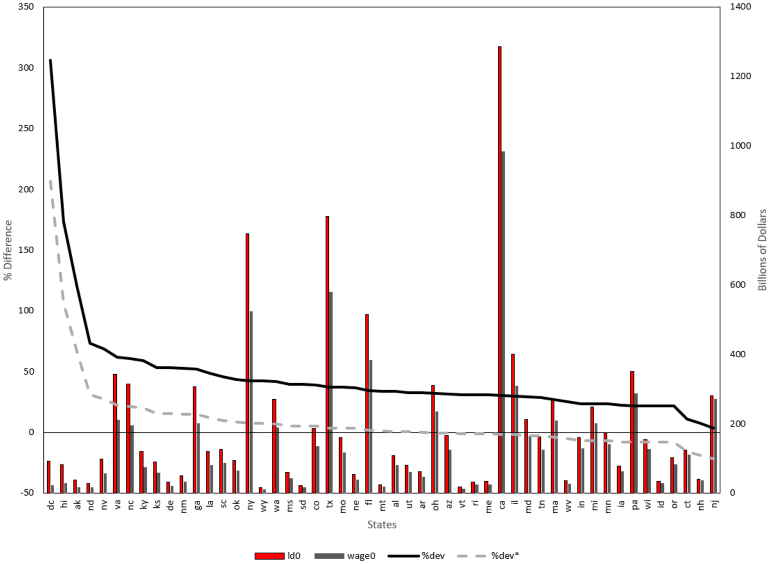
Savings:

- Reference savings rates on upper income groups based on Zucman and Saez (2016 – QJE) – find that bottom 90% of wealth in US have a 0% savings rate, where household wealth is roughly 400% of national income.
- Assume zero savings for income groups less than $100k.

Transfers:

- Per the CBO, transfers minus tax payments are negative for income groups above 100k (and positive for lower income groups).
- Assume that observed transfer payments in SOI are an upper bound for higher income groups and a lower bound for lower income groups to capture *net* transfers.

Example: reconciling labor demands



Calibrated income shares

	Total Income	Income Trn	Income Cap	Income Lab	Expend Cons	Expend Save	Expend Taxes
<10k	473.87	0.38	0.22	0.41	0.98		0.02
10_25k	874.77	0.11	0.28	0.60	0.97		0.03
25_50k	1903.79	0.07	0.22	0.71	0.94		0.06
50_75k	1852.82	0.08	0.26	0.66	0.92		0.08
75_100k	1695.83	0.09	0.29	0.63	0.90		0.10
100_200k	4171.68	0.02	0.36	0.62	0.72	0.14	0.13
200_500k	2968.05	0.01	0.46	0.53	0.56	0.24	0.20
500k_1m	1282.91	0.00	0.57	0.42	0.39	0.35	0.26
>1m	3271.03	0.00	0.80	0.20	0.38	0.35	0.28

Note that regional heterogeneity exists at the state level.

Future data work

- Expenditure data.
- Linking CPS data with SOI to gain a fuller perspective on transfer income.
    - Reconcile “tax-filing units” with households.
    - Not everyone files income taxes.
    - Non-taxed transfers.

Static vs. steady state calibration

Initial application of the calibration routine was in a *static* context. In a *dynamic* Ramsey framework, investment levels must be adjusted up to achieve steady state levels.

- Stark contrast between investment in IO tables and steady state levels (roughly 2x IO levels under reasonable growth assumptions).
- Traditional treatment of this problem has been to move some consumption demand into savings.
- Impacts the household recalibration routine. Inflates levels of savings and generates implausible consumption levels for upper income groups.

Redefined 2 step procedure:

- Generate household decomposition with imposed steady state levels of investment. Note – this inflates transfers and savings, illustrating issues with national accounts and steady state assumptions.
- Assume additional investment demand in the steady state comes out of intermediate demand rather than consumption.

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```
Nonnegative Variables
*$SECTORS:
        Y        Production
        A        Armington composite
        M        Imports
        X        Exports

*$COMMODITIES:
        PD       Domestic price index
        PX       Export price index
        PM       Import price index
        PA       Armington price index
        PL       Wage rate index
        RK       Rental price index
        PFX      Foreign exchange

*$CONSUMERS:
        HH       Private households
        GOVT     Government

*$AUXILIARY:
        TAU      Replacement tax;
```

Define equations

```
marketd..  Y*DY =e= A*DA;
profity..  PKL*(ld0*pl0 + kd0*rr0) =g= PY*(d0+x0*px0);

marketa..  A*a0 =g= GOVT/PA + C + i0;
profita..  PDM*(m0*pm0 + d0) =e= PA*a0*(1-ta);

marketm..  M*m0 =e= A*MA;
profitm..  PFX*pwm =e= PM;

marketx..  Y*XY =e= X*x0;
profitx..  PX =e= PFX*pwx;

marketfx.. X*pwx*x0 - M*pwm*m0 =e= -bopdef ;

marketk..  kd0    =e= Y*KD;
marketl..  ld0+l0 =e= Y*LD + L;

incomeg..  GOVT =e= PFX*bopdef + PA*dtax +  PA*g0*TAU
           + tx*PX*XY*Y + tk*RK*KD*Y + tl*PL*LD*Y +  tm*PM*MA*A + ta*PA*a0*A;
taudef..   GOVT =e= PA * g0;

incomeh..  HH =e= PL*(ld0+l0) - PA*dtax - PA*g0*TAU + RK*kd0 - PA*i0  ;

model mcp123 /marketd.PD, marketa.PA, marketm.PM, marketx.PX,
              marketfx.PFX, marketk.RK, marketl.PL,
              profity.Y,  profita.A,  profitm.M,  profitx.X,
              incomeg.GOVT, incomeh.HH, taudef.TAU/;
```

Three Alternative Recursive Models

1. Constant marginal propensity to save: This is a closed economy model, so savings equals investment. In this model, we have a constant fraction of income allocated to savings:

$$S = I = \alpha \frac{M}{P_I}$$

2. The Ballard-Fullerton-Shoven-Whalley closure.
   The price of "savings" is defined as the ratio of the cost of new capital (P_I) and the real rental price of capital R_K/P_C:

$$P_S = \frac{P_I P_C}{R_K}$$

$$P_C = \left(\theta P_A^{1-\sigma} + (1-\theta) P_\ell^{1-\sigma}\right)^{1/(1-\sigma)}$$

$$S = \left(\frac{PC}{P_S}\right)^{\sigma_S} C$$

3. The Monash closure. A logistic curve relates the rate of capital accumulation to the real rate of return to capital (investment is unaffected by the capital price).

```
sdef..     S =E=  (KGR+delta)/(gr+delta) * kstock/k0;
kgrdef..   RK/PK * ir =e= ror0 + (1/csmurf) * ( (log(KGR-kgmin)     - log(kgmax-KGR)) -
                                               (log(trendk-kgmin)  - log(kgmax-trendk)) );
```

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The Ramsey model is often presented as an *infinite-horizon* dynamic optimization problem:

$$\max U(C) = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t \frac{C_t^{1-\theta} - 1}{1-\theta} \tag{1}$$

$$\begin{aligned} \text{s.t.} \qquad C_t &= f(K_t) - I_t \\ K_{t+1} &= (1-\delta)K_t + I_t \\ K_0 &= \bar{K}_0 \\ I_t &\geq 0 \end{aligned}$$

where $f'(K) > 0$ and $f''(k) < 0$.

CIES Preferences are CES Preferences

The maximand in the dynamic optimization model is a montonic transformation of conventional CES utility function:

$$\hat{U}(C) = \left[\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t C_t^{1-\theta} \right]^{1/1-\theta}$$

Note that

$$\hat{U} = \mathcal{V}(U) = [aU + \kappa]^{1/a}$$

where

$$\kappa = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t = \frac{1+\rho}{\rho},$$

and

$$a = 1 - \theta.$$

Hence, $\mathcal{V}()$ is a monotonic transformation $(\mathcal{V}' > 0)$

Consumer subproblem for the Ramsey model:

$$\max \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^{t} U(C_t)$$

s.t.

$$\textstyle\sum_{t=0}^{\infty} p_t C_t = p_0^K \bar{K}_0 + \sum_{t=0}^{\infty} p_t^L \bar{L}_t$$

Define:

$$A_T^* = \sum_{t=T+1}^{\infty} \left(p_t c_t^* - p_t^L \bar{L}_t \right)$$

Approximating an Infinite Horizon

In steady-state:

$$A_T^* = (p_T C_T - p_T^L L_T) \sum_{k=0}^{\infty} \left(\frac{1+g}{1+r} \right)^k = (p_T C_T - p_T^L L_T) \frac{1+r}{r-g}$$

Then consider the equivalent *pair* of model:

$$\max \sum_{t=0}^{T} \left(\frac{1}{1+\rho} \right)^t U(C_t)$$

s.t.

$$\textstyle\sum_{t=0}^{T} p_t C_t = p_0^K \bar{K}_0 + \sum_{t=0}^{T} p_t^L \bar{L}_t - A_T$$

$$\max \sum_{t=T+t}^{\infty} \left(\frac{1}{1+\rho} \right)^t U(C_t)$$

s.t.

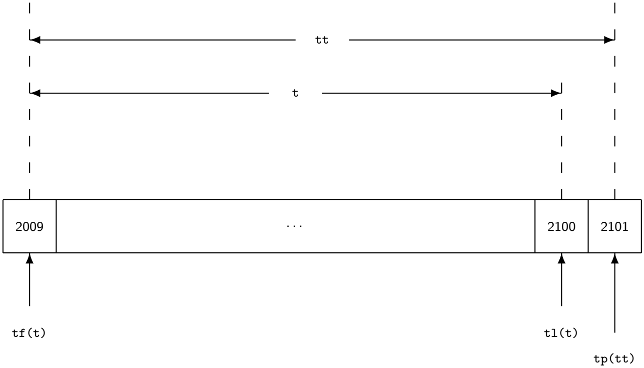
$$\textstyle\sum_{t=T+1}^{\infty} p_t C_t = A_T + \sum_{t=T+1}^{\infty} p_t^L \bar{L}_t$$

In a complementarity format, the post-terminal capital stock can be *targetted* to provide a tight approximation of the infinite-horizon path:

$$I_T/I_{T-1} = 1 + g \quad \perp K_{T+1}$$

N.B. In a model with multiple infinitely-lived agents (for example, in a multi-region Ramsey model) we have to use A^*_{rT} to ascertain ownership shares of the terminal capital stocks.

Time Structure and Dynamic Sets



`tt`	Time horizon (with the first year of the post-terminal period)
`t(tt)`	Time period over the model horizon
`tf(tt)`	First period of the model (2009)
`tl(tt)`	Last endogenous period (2100)
`tp(tt)`	Post-terminal period (the element of set `tt` which is not an element of `t`, 2101)

The Ramsey Model as an Equilibrium Problem

1. Market clearance conditions and associated market prices:

   Output market (market price p_t):

   $$Y_t = C_t(p, M) + I_t$$

   Labor market (wage rate p_t^L):

   $$\bar{L}_t = a_L(r_t^K(1 + \tau_t), p_t^L)\ Y_t$$

   Market for capital services (capital rental rate at producer prices – gross of tax – is $r_t^K(1 + \tau_t)$):

   $$K_t = a_K(r_t^K(1 + \tau_t), p_t^L)\ Y_t$$

   Capital stock (capital purchase price p_t^K):

   $$K_{t+1} = (1 - \delta)K_t + I_t$$

The Ramsey Model as an Equilibrium Problem (cont)

2. Zero profit conditions:

   Output (Y_t):

$$p_t = c(p_t^L, r_t^K)$$

   Investment ($I_t \geq 0$):

$$p_t \geq p_{t+1}^K$$

   Capital stock (K_t):

$$p_t^K = r_t^K + (1-\delta)p_{t+1}^K$$

3. Income balance:

$$M = p_0^K \bar{K}_0 + \sum_{t=0}^{\infty} p_t^L \bar{L}_t$$

4. Terminal approximation:

$$\frac{I_T}{I_{T-1}} = 1 + g$$

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Benchmark replication is a common strategy for evaluating logical consistency of a static general equilibrium model. The idea is that if a model has been parameterized on the basis of economic transactions in a reference equilibrium with policy parameters $\bar{\tau}$.
In the dynamic setting a similar consistency check is possible. If a growth model is calibrated to a *steady-state* growth path, this can serve the same role as the benchmark equilibrium dataset in a static model.

The zero-profit condition for I_t reveals the price level for capital:

$$p^K_{t+1} = \frac{p^K_t}{1+\bar{r}} = p_t$$

hence

$$p^K_t = (1+\bar{r})p_t$$

The base year price of capital is then:

$$\bar{p}^K = 1 + \bar{r}$$

The zero profit condition for K_t determines the price level for r_t^K:

$$p_t^K = r_t^K + (1 - \delta) p_{t+1}^K$$

Substituting the values of p_t^K and p_{t+1}^K reveals that the base year rental price of capital is sufficient to cover interest plus depreciation:

$$\bar{r}^K = \bar{r} + \delta$$

Finally, consider the market clearance condition for capital in the first period:

$$K_1 = \bar{K}_0(1-\delta) + \bar{I} = (1+g)\bar{K}_0$$

This implies that base year investment can be calculated on the basis of growth and depreciation of the base year capital stock:

$$\bar{I} = \bar{K}_0(g+\delta)$$

We then can use $\bar{r}^K$ to determine $\bar{K}_0$ on the basis of the value of capital earnings in the base year, V^K, hence:

$$\bar{I} = V^K \; \frac{g + \delta}{r + \delta}$$

The problem that arises in applied models is that $\bar{I}$ and V^K will not satisfy this relation for arbitrary values of $\bar{g}$, $\bar{r}$ and δ. Part of the art of economic equilibrium analysis involves reconciliation of theory and data. It is perhaps not surprising that when we add more theory (e.g., intertemporal decisions) we introduce more constraints on the underlying database.

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Activities and Arbitrage Conditions

```
$ontext
$model:dynamic

$sectors:
        Y(r,s,t)                !       Production
        X(r,g,t)                !       Disposition
        A(r,g,t)                !       Absorption
        C(r,h,t)                !       Household consumption
        MS(r,m,t)               !       Margin supply
        K(r,g,t)                !       Sectoral capital stock
        I(r,g,t)                !       Sectoral investment
        INV(r,t)                !       Aggregate investment
```

```
$commodities:
        PA(r,g,t)        !        Regional market (input)
        PY(r,g,t)        !        Regional market (output)
        PD(r,g,t)        !        Local market price
        RK(r,s,t)        !        Sectoral capital rental rate
        PM(r,m,t)        !        Margin price
        PC(r,h,t)        !        Consumer price index
        PN(g,t)          !        National market
        PL(r,t)          !        Regional wage rate
        PK(r,g,tt)       !        Capital purchase price
        PINV(r,t)        !        Cost of new vintage investment
        PKNYSE           !        Price index for extant capital
        PFX(t)           !        Foreign exchange
```

Households/Budget and Auxiliary Constraints

```
$consumer:
    GOVT(t)                          ! Aggregate government
    NYSE                             ! Aggregate capital owner
    RIH(r,h)$saving(h)               ! Representative intertemporal household
    RSH(r,h,t)$subsistence(h)        ! Representative subsistence household

$auxiliary:
        TAXRATE(t)                     ! Replacement tax
        KT(r,g)                        ! Terminal capital stock
```

Intra-Period Production

```
$prod:Y(r,s,t)$y_(r,s)  s:0 va:1
        o:PY(r,g,t)     q:ys0(r,s,g)           a:GOVT(t)  t:ty0(r,s)       p:(1-ty0(r,s))
        i:PA(r,g,t)     q:id0(r,g,s)
        i:PL(r,t)       q:ld0(r,s)     va:
        i:RK(r,s,t)     q:kd0(r,s)     va:

$prod:X(r,g,t)$x_(r,g)  t:4
        o:PFX(t)          q:(x0(r,g)-rx0(r,g))  p:pref(t)
        o:PN(g,t)         q:xn0(r,g)            p:pref(t)
        o:PD(r,g,t)       q:xd0(r,g)            p:pref(t)
        i:PY(r,g,t)       q:s0(r,g)

$prod:A(r,g,t)$a_(r,g)  s:0 dm:4  d(dm):2
        o:PA(r,g,t)       q:a0(r,g)             a:GOVT(t) t:ta0(r,g)      p:(1-ta0(r,g))
        o:PFX(t)          q:rx0(r,g)
        i:PN(g,t)         q:nd0(r,g)      d:                              p:pref(t)
        i:PD(r,g,t)       q:dd0(r,g)      d:                              p:pref(t)
        i:PFX(t)          q:m0(r,g)       dm:   a:GOVT(t) t:tm0(r,g)      p:(pref(t)*(1+tm0(r,g)))
        i:PM(r,m,t)       q:md0(r,m,g)                                    p:pref(t)

$prod:MS(r,m,t)
        o:PM(r,m,t)       q:(sum(gm, md0(r,m,gm)))
        i:PN(gm,t)        q:nm0(r,gm,m)
        i:PD(r,gm,t)      q:dm0(r,gm,m)

$prod:C(r,h,t)    s:1
        o:PC(r,h,t)       q:c0_h(r,h)
        i:PA(r,g,t)       q:cd0_h(r,g,h)        p:pref(t)

$prod:INV(r,t)
        o:PINV(r,t)             q:(sum(g,kn0(r,g)))
        i:PA(r,g,t)             q:i0(r,g)
```

```
$prod:K(r,g,tt)$t(tt)
        o:PK(r,g,tt+1)          q:(k0(r,g)*(1-delta(r,g)))
        o:RK(r,g,tt)            q:kd0(r,g)
        i:PK(r,g,tt)            q:k0(r,g)

$prod:I(r,g,tt)$t(tt)
        o:PK(r,g,tt+1)          q:kn0(r,g)
        i:PINV(r,tt)            q:kn0(r,g)
```

```
*       Government budget balance in every period:

$demand:GOVT(t)
    d:PA(r,g,t)     q:g0(r,g)
    e:PFX(t)        q:(qref(t)*govdef0)     ! q:gdef0(t)
    e:PL(r,t)       q:(sum(h,tx(r,h)*ls0(r,h,t)))           r:TAXRATE(t)
    e:PINV(r,t)     q:(sum(h,tx(r,h)*qref(t)*ke0(r,h)))     r:TAXRATE(t)

*       Capital markets are fully integrated:

$demand:NYSE
        d:PKNYSE
        e:PY(r,g,t)     q:(qref(t)*yh0(r,g))
        e:PK(r,s,t0)    q:k0(r,s)
        e:PK(r,g,tp)    q:(-ktref(r,g))         R:KT(r,g)
```

```
*       Saving households are dynastic allocation consumption expenditure

$demand:RIH(r,h)$saving(h)  s:sigma_t(h)
        d:PC(r,h,t)     q:(qref(t)*c0_h(r,h))           p:pref(t)
        e:PFX(t)        q:(qref(t)*trn0(r,h))
        e:PL(r,t)       q:ls0(r,h,t)
        e:PL(r,t)       q:(-tx(r,h)*ls0(r,h,t))         r:TAXRATE(t)
        e:PINV(r,t)     q:(-tx(r,h)*qref(t)*ke0(r,h))   r:TAXRATE(t)
        e:PKNYSE        q:ks0(r,h)

*       Subsistence households operate on a cash basis in every period.

$demand:RSH(r,h,t)$subsistence(h)
        d:PC(r,h,t)     q:(qref(t)*c0_h(r,h))           p:pref(t)
        e:PFX(t)        q:(qref(t)*trn0(r,h))
        e:PL(r,t)       q:ls0(r,h,t)
        e:PL(r,t)       q:(-tx(r,h)*ls0(r,h,t))         r:TAXRATE(t)
        e:PINV(r,t)     q:(-tx(r,h)*qref(t)*ke0(r,h))   r:TAXRATE(t)
        e:PKNYSE        q:(ks0(r,h)*thetas(r,h,t))
```

```
$constraint:TAXRATE(t)

  GOVT(t) =e= sum((r,g),PA(r,g,t)*qref(t)*g0(r,g));

$constraint:KT(r,g)

  sum(t$tl(t+1), I(r,g,t+1)/I(r,g,t) -

                      Y(r,g,t+1)/Y(r,g,t)) =E= 0;
```