Adding scale economies and imperfect competition to general equilibrium

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(Chapter 9 of my grad course)

An introduction to Dixit-Stiglitz CES preferences

D-S preferences are a special, symmetric case of CES preferences, elasticity of substitution > 1.

Y will be a competitive, constant-returns industry while X will consist of an endogenous number of differentiated varieties.

Utility of the representative consumer in each country is Cobb-Douglas, and the symmetry of varieties within a group of goods allows us to write utility as follows ($0 < \alpha < 1$).

$$U = X_c^{\beta} Y^{1-\beta}, \qquad X_c \equiv \left[\sum_i^N (X_i)^{\alpha} \right]^{1/\alpha}$$

where the number of varieties N is endogenous.

This function permits the use of two-stage budgeting, in which the consumer first allocates total income (M) between Y and X_c.

Let e denote the minimum cost of buying one unit of X_c at price p for the individual varieties (i.e., e is the unit expenditure function for X_c). Y is numeraire. First-stage budgeting yields:

$$Y = (1-\beta)M \qquad X_c = \beta M/e$$

$$e(p^k) = \min(X_i) \sum_i pX_i \quad st \quad X_c = 1$$

Let $M_x = \beta M$ be the expenditure on X in aggregate. Solve for the demand for a given X variety, and for the price index e.

$$X_i = p_i^{-\sigma}\left[\sum p_j^{1-\sigma}\right]^{-1} M_x \qquad \sigma = \frac{1}{1-\alpha}, \qquad \alpha = \frac{\sigma - 1}{\sigma}$$

$$e = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$ if all prices equal: $e = N^{\frac{1}{1-\sigma}} p$

$$X_i \equiv p_i^{-\sigma} e^{\sigma - 1} M_x \qquad since \qquad e^{\sigma - 1} = \left[\sum p_j^{1-\sigma}\right]^{-1}$$

From pre-course notes: Firm's perceived price elasticity of demand assuming M is fixed and (by Cobb-Douglas) therefore M_x is fixed.

Notation:

s = firm's market share

σ = elasticity of substitution among varieties within a sector

η = firm's perceived price elasticity

Cournot: firm views other firms *outputs* and *expenditure* M_x as fixed.

$$\eta_c = \frac{1}{s + (1-s)\frac{1}{\sigma}} = \frac{\sigma}{\sigma s + (1-s)}$$

Bertrand: firm views other firms *prices* and *expenditure* M_x as fixed.

$$\eta_b = \sigma - s(\sigma - 1) = (1-s)\sigma + s$$

Special cases:

at s = 0, $\eta_c = \eta_b = \sigma$ large group monopolistic competition

at s = 1, $\eta_c = \eta_b = 1$ monopoly

for 0 < s < 1, $\eta_c < \eta_b$ Cournot less elastic than Bertrand for $0 < s < 1$, $\sigma = \infty$, $\eta_c = \dfrac{1}{s}$, $\eta_b = \infty$ perfect substitutes =>

Bertrand approaches perfect competition
Cournot elasticity approaches firm's inverse of market share

From perceived elasticities of demand to markups

Suppose demand for good X_i is just written in inverse form $p_i(X_i)$ so the monopolist's revenue is $R_i = p_i(X_i)X_i$. Note that this inverse demand function is defined differently for Cournot and Bertrand.

For Cournot, it is how a firm's price responds to its own output holding outputs of other firms constant.

For Bertrand, it is how a firm's price responds to its own output holding the prices of other firms constant.

Income is perceived as constant in both cases.

$$\frac{\partial R_i}{\partial X_i} = p_i + X_i \frac{\partial p_i}{\partial X_i} = p_i + p_i \left[\frac{X_i}{p_i} \frac{\partial p_i}{\partial X_i} \right] = p_i \left[1 - \frac{1}{\eta_i} \right]$$

$$\eta_{ib} \equiv - \left[\frac{p_i}{X_i} \frac{\partial X_i}{\partial p_i} \right]_{\bar{p}_{j \neq i}} \qquad \eta_{ic} \equiv - \left[\frac{p_i}{X_i} \frac{\partial X_i}{\partial p_i} \right]_{\bar{X}_{j \neq i}}$$

$$MR_i = p_i(1 - mk_i) \qquad mk_i = \frac{1}{\eta_i}$$

where *mk* is the optimal markup what is referred to as a gross basic and a bar indicates that a variable is held constant.

Often in the equation price equals marginal cost, the markup is inverted to other side of the equation. So we may see it written as either

$$p\left[1 - \frac{1}{\eta}\right] \equiv mc \qquad p = \left[\frac{\eta}{\eta - 1}\right]mc$$

Note again that, in large-group monopolistic-competition, the price elasticity reduces to just the elasticity of substitution σ for both the Cournot and Bertrand cases as noted above.

Also note that the formulas depend on the assumption that the X composite and Y (upper "nest" of the function) are Cobb-Douglas substitutes; σ = 1 for the upper nest.

If the upper nest is not CD, then the formulas are more complicated, involving both the within X_c and between X_c and Y elasticities of substitution.

5.4 Monopolistic-competition I: large group with D-S CES

The assumption in “large-group” monopolistic competition is that there are many firms: individual firms view e, M as *constants*.

Thus the elasticity of demand for an individual variety is just σ.

Equilibrium in the X sector involves two equations in two unknowns. The unknowns are X, output per variety and N, the numbers of varieties or firms.

The two equations are the firm’s optimization condition, marginal revenue equals marginal cost, and the free-entry or zero profit condition, prices equals average cost.

Gains from increased final and intermediate goods variety.

Total income is given by L when the wage is chosen as numeraire.

Symmetry I: all X goods are imperfect but symmetric substitutes

Symmetry I: all X goods have the same cost function

Symmetry III: fixed and marginal costs have the same functional form: f/c is a constant.

X and p_x will denote the price of a representative good which are the same for all goods actually produced

$$U = \left[\sum_i X_i^{\alpha}\right]^{\frac{\beta}{\alpha}} Y^{1-\beta} \qquad \sigma = \frac{1}{1-\alpha} \qquad L = np_x X + p_y Y \tag{1}$$

the consumer's demands for *X* varieties and *Y* are

$$Y = (1-\beta)\frac{L}{p_y} \qquad X_i = p_{xi}^{-\sigma}\left[\sum_i p_{xi}^{1-\sigma}\right]^{-1}\beta L \qquad nX = \beta\frac{L}{p_x} \tag{2}$$

The variety's own price appears both as the first term on the right-hand side of the second equation of (2) but also appears in the summation term inside the square brackets.

The effect of a change in a firm's price on the summation term in square brackets become extremely small as the number of varieties (firms) n becomes large.

Assumes that an individual firm is too small to affect the summation term in (2), an assumption known as "large-group monopolistic competition.

The price elasticity of demand for an individual good is given simply by σ, the elasticity of substitution among the *X* goods

$$-\frac{p_x}{X}\frac{\partial X}{\partial p_x} = \sigma \qquad mr_x = p_x(1 - 1/\sigma) = mc_x \tag{3}$$

Inequality	Definition	Complement Var	
$p_x(1 - 1/\sigma) \leq mc_x$	pricing for *X*	*X*	(4)
$(p_x/\sigma)X \leq fc_x$	pricing for *n* (free entry)	*n*	(5)
$p_y \leq mc_y$	pricing for *Y*	*Y*	(6)

Then there are three market-clearing conditions, which require that supply equal demand (strictly speaking supply is greater than or equal to demand)

$$(1-\beta)L/p_y \;\le\; Y \qquad \text{demand/supply } Y \qquad p_y \tag{7}$$

$$\beta L/p_x \;\le\; nX \qquad \text{demand/supply } X \text{ varieties} \qquad p_x \tag{8}$$

$$(mc_y)Y + n(mc_x)X + n(fc_x) \;=\; L \qquad \text{demand/supply } L \qquad w \tag{9}$$

Equations (4) and (5) can be solved for both X and p_x. Then these solution values can be used in (8) to get n.

$$X = (\sigma - 1)\frac{fc_x}{mc_x} \qquad n = \frac{\beta L}{\sigma fc_x} \qquad nX = \frac{(\sigma - 1)}{\sigma}\frac{\beta L}{mc_x} \tag{10}$$

The output of any good that is produced is a constant and that any expansion in the economy creates a proportional increases in variety n.

Let *X/L*, the consumption of a representative variety per capita, be given by *C*. Then note from the last equation of (10) that *nC* is a constant:

$$C = \frac{X}{L} = \frac{(\sigma - 1)}{\sigma}\frac{\beta}{mc_x}\frac{1}{n} \equiv \frac{\gamma}{n} \tag{11}$$

$$U_x = \left[nC^{\alpha}\right]^{\frac{1}{\alpha}} = n^{\frac{1}{\alpha}}C = n^{\frac{1-\alpha}{\alpha}}\gamma = n^{\frac{1}{\sigma-1}}\gamma = \left[\frac{\beta L}{\sigma fc_x}\right]^{\frac{1}{\sigma-1}}\gamma$$

The per-capita value of (composite) X consumption increases with the size of the economy. This is a pure variety effect: Utility increases when a consumer gets half as much of each of twice as many goods.

Markets	Production Sectors C	FC	Y	W	Consumers CONS	ENTR
PX	100			-100		
PY			100	-100		
PN		20				-20
PW				200	-200	
PL	-80	-20	-100		200	
MK	-20					20

There are a number of ways to organize the benchmark data, this is one of them.

Markup revenues (MK) are not directly observed by IO economists have techniques for estimating these.

I introduce an artificial or "dummy" agent ENTR (entrepreneur). ENTR receives the markup revenues and "demands" fixed costs.

In equilibrium, the total value of fixed costs produced equals markup revenues, which is a way of modeling the free-entry zero-profit condition.

The activity level for N (production of fixed costs) corresponds to the number of varieties produced in equilibrium, and so affects the price index and welfare.

marginal revenue = mc price = average cost

$$p(1 - 1/\sigma) = p(1 - mk) = mc \qquad p = mc + fc/X$$

Subtracting the second equation from the first:

$$p(1 - mk) - p = mc - mc - fc/X$$

$$p(mk)X = fc$$ markup revenues = fixed costs.

The counter-factual experiment doubles the size of the economy.

The X sector's output is homogeneous of degree 1.25 in factor inputs with $\sigma = 5$, if by X sector's output here we mean X_c.

The X sector expands only through the entry of new firms, the output of a representative firm, X, is constant. X_c is given by

$$X_c = \left[N X^{\alpha} \right]^{\frac{1}{\alpha}} = N^{\frac{1}{\alpha}} X = N^{\frac{\sigma}{\sigma - 1}} X = N^{1.25} X$$

Double the size of the economy. Per-capita effect.

$$X_{cpcap} = (2 N_0)^{1.25} (X_0 / 2) = 2^{0.25} N_0^{1.25} X_0$$

```
PRICEX..   PL =G= PX*(1-1/SI);

PINDEX..   (N*PX**(1-SI))**(1/(1-SI)) =G= PE;

PRICEN..   PL =G= PN;

PRICEY..   PL =G= PY;

PRICEW..   (PE**0.5)*(PY**0.5) =G= PW;


DX..       X*80  =G=  PX**(-SI)*(PE**(SI-1))*CONS/2;

DN..       N*FC  =G= (PX/SI)*X*80*N/PN;

DY..       Y*100 =G= CONS/(2*PY);

DW..       200*W =G= (1.25**0.5)*CONS/PW;

LAB..      ENDOWL =E= Y*100 + N*X*80 + N*FC;

INCOME..   CONS =E= PL*ENDOWL;
```

5.5 Monopolistic-competition II: small group (kyiv13.gms)

This file calibrates the data to the Bertrand markup rule. After solving the model, the markup rule is changed to the Cournot case.

There is no change to the elasticity of substitution. Therefore, the Cournot case is not re-calibrated to yield the same benchmark solution.

Switching to the Cournot formula raises the initial markup and number of firms and lowers welfare some.

Increasing the size of the economy increases welfare in both cases. In both cases, this is a combination of increased variety and lower markups (increased output per firm).

To calibrate to the same data as in KYIV12.GMS (Bertrand large-group MC),

We use sigma = 6.3333, instead of sigma = 5.

markup = 1/(sigma - (1/(1+N))(sigma - 1) = 0.20

Two equations for markup are specified, two different model declaration.

```
MARKUPB..  MK =E= 1/(SI - 1/N*(SI - 1));
MARKUPC..  MK =E= (1/N) + (1-(1/N))/SI;


MODEL M75B /PRICEX.X, PRICEY.Y, PRICEW.W, PRICEN.N, PINDEX.PE,
           DX.PX, DN.PN, DY.PY, DW.PW,
           LAB.PL, MARKUPB.MK, INCOME.CONS/;

MODEL M75C /PRICEX.X, PRICEY.Y, PRICEW.W, PRICEN.N, PINDEX.PE,
           DX.PX, DN.PN, DY.PY, DW.PW,
           LAB.PL, MARKUPC.MK, INCOME.CONS/;
```

```
LOOP(I,
LOOP(J,

SIZE(I) =  5.2 - 0.2*ORD(I);
ENDOWL = 200*SIZE(I);

IF(ORD(J) EQ 1,
SOLVE M75B USING MCP;
ELSE SOLVE M75C USING MCP;);

WELFARE(I,J) = W.L;
WELFCAP(I,J) = WELFARE(I,J)/SIZE(I);
MARKUPS(I,J) = MK.L;
NUMBERF(I,J) = N.L;

);
);
RESULTS1(I, "SIZE") = SIZE(I);
RESULTS1(I, "WELFCAP-B") = WELFCAP(I, "J1");
RESULTS1(I, "WELFCAP-C") = WELFCAP(I, "J2");
RESULTS1(I, "NUMBERF-B") = NUMBERF(I, "J1");
RESULTS1(I, "NUMBERF-C") = NUMBERF(I, "J2");
RESULTS1(I, "MARKUP-B")  = MARKUPS(I,"J1");
RESULTS1(I, "MARKUP-C")  = MARKUPS(I,"J2");
```

```
DISPLAY RESULTS1;

* Write parameter RESULTS to an Excel file madison7.xlsx,
* starting in Sheet1, cell A3


Execute_Unload 'madison7.gdx' RESULTS1
execute 'gdxxrw.exe madison7.gdx par=RESULTS1 rng=SHEET1!A3:I29'
```

Kyiv13.gms Comparing Bertrand (B) and Cournot (C) as economy grows: Calibrated for Bertrand

	SIZE	WELFCAP-B	WELFCAP-C	NUMBERF-B	NUMBERF-C	MARKUP-B	MARKUP-C
I1	5	1.167	1.163	16.632	20.000	0.166	0.200
I2	4.8	1.162	1.158	16.000	19.338	0.167	0.201
I3	4.6	1.158	1.153	15.368	18.675	0.167	0.203
I4	4.4	1.153	1.148	14.737	18.010	0.167	0.205
I5	4.2	1.148	1.143	14.105	17.342	0.168	0.206
I6	4	1.142	1.137	13.474	16.672	0.168	0.208
I7	3.8	1.137	1.131	12.842	16.000	0.169	0.211
I8	3.6	1.131	1.125	12.211	15.325	0.170	0.213
I9	3.4	1.125	1.118	11.579	14.647	0.170	0.215
I10	3.2	1.119	1.111	10.947	13.965	0.171	0.218
I11	3	1.112	1.104	10.316	13.279	0.172	0.221
I12	2.8	1.105	1.096	9.684	12.588	0.173	0.225
I13	2.6	1.097	1.088	9.053	11.893	0.174	0.229
I14	2.4	1.089	1.078	8.421	11.191	0.175	0.233
I15	2.2	1.080	1.068	7.789	10.482	0.177	0.238
I16	2	1.070	1.057	7.158	9.765	0.179	0.244
I17	1.8	1.059	1.044	6.526	9.038	0.181	0.251
I18	1.6	1.047	1.030	5.895	8.299	0.184	0.259
I19	1.4	1.034	1.014	5.263	7.546	0.188	0.269
I20	1.2	1.018	0.995	4.632	6.773	0.193	0.282
I21	1	1.000	0.972	4.000	5.976	0.200	0.299
I22	0.8	0.978	0.943	3.368	5.145	0.211	0.322
I23	0.6	0.948	0.903	2.737	4.264	0.228	0.355
I24	0.4	0.904	0.841	2.105	3.303	0.263	0.413
I25	0.2	0.809	0.713	1.474	2.178	0.368	0.545

9.3 Monopolistic competition with horizontal multinationals

Partial-equilibrium models give good insight, but have limitations from the point of view of trade theory and policy.

(1) no role for factor prices and factor endowments, no reverse effect of the introduction of mnes on factor prices.

(2) no role for entry and exit in response to liberalization.

In this section, we study how endogenous multinational firms are introduced in a general-equilibrium context.

Start with two sectors, one factor: monopolistic-competition, national and horizontal (2-plant) firms.

$$U = \left[\sum_i X_i^{\alpha}\right]^{\frac{0.5}{\alpha}} Y^{0.5} \qquad \sigma = \frac{1}{1-\alpha} \qquad L = np_x X + p_y Y$$

If you solve the optimization problem, the consumer's demands for *X* varieties and *Y* are

$$Y = \frac{L}{2p_y} \qquad X_i = p_i^{-\sigma}\left[\sum_i p_i^{1-\sigma}\right]^{-1}\frac{L}{2} \qquad nX = \frac{L}{2p_x}$$

Marginal cost Y, marginal cost X, and fixed costs of an X variety:

$mc_y \qquad mc_x \qquad fc_x$

Large group monopolistic competition: firms view [ ] as fixed, so demand for an individual variety is iso-elastic

marginal revenue is given by $p_x(1 - 1/\sigma)$

Autarky equilibrium is given as the solution to:

Inequality	Definition	Complementary Variable
$p_y \le mc_y$	pricing for *Y*	*Y*
$p_x(1 - 1/\sigma) \le mc_x$	pricing for *X*	*X*
$(p_x/\sigma)X \le fc_x$	pricing for *n* (free entry)	*n*
$L/(2p_y) \le Y$	demand/supply *Y*	p_y
$L/(2p_x) \le nX$	demand/supply *X* variety	p_x
$(mc_y)Y + n(mc_x)X + n(fc_x) = L$	demand/supply *L*	*w*

This model can be solved analytically and yields:

$$X = (\sigma - 1)\frac{fc_x}{mc_x} \qquad n = \frac{L}{2\sigma fc_x} \qquad nX = \frac{(\sigma - 1)}{\sigma}\frac{L}{2mc_x}$$

Now suppose that, while trade is prohibitive, each firm can establish a second plant in the other country for an additional fixed cost.

The fixed cost for a two-plant firm is given by βfc_x , $2 > \beta > 1$.

Multi-plant economies of scale due to non-rivaled nature of knowledge capital.

Replace fc_x with βfc_x and replace L with $2L$; the *total* two-country output of an X variety and *total* varieties are now:

$$X = (\sigma - 1)\beta \frac{fc_x}{mc_x} \qquad n = \frac{L}{\beta \sigma fc_x}$$

Each country gets half of each X variety: *single-country totals* are

$$X = \frac{(\sigma - 1)}{2} \beta \frac{fc_x}{mc_x} \qquad n = \frac{L}{\beta \sigma fc_x} \qquad nX = \frac{(\sigma - 1)}{\sigma} \frac{L}{2\,mc_x}$$

Note that *nX* is the same as it was in the domestic-firm case:

$$U = (nX^{\alpha})^{\frac{0.5}{\alpha}} Y^{0.5} = (n^{1-\alpha} n^{\alpha} X^{\alpha})^{\frac{0.5}{\alpha}} Y^{0.5} = n^{\frac{1-\alpha}{\alpha}0.5} \left[(nX)^{0.5} Y^{0.5} \right]$$

The term in square bracket on the right-hand side is unchanged with multinationals.

Denoting the autarky value of n as n_a and then since the new value is $n_m = (2/\beta)n_a$, then the ratio of utility in the multinational regime to autarky is given by

$$\frac{U_m}{U_a} = (n_m/n_a)^{\frac{1-\alpha}{\alpha}0.5} = (2/\beta)^{\frac{1-\alpha}{\alpha}0.5} > 1$$

If β = 1.5 and α = 0.75 (an elasticity of substitution of 4 between *X* varieties), then this ratio is 1.05:

there is a 5 percent gain in per-capita welfare (10 percent gain in utility from X) from introducing horizontal multinationals.

Horizontal multinationals improve welfare by exploiting firm-level scale economies; that is, the non-rivaled property of knowledge-based assets.

Return to the domestic-firm case, and assume that X can be traded at the (gross) trade cost t (t = 1 + (iceberg melt rate)).

Returning to the utility function, the demand for an individual domestic variety can be re-written using the price index e:

$$X_i = p_i^{-\sigma} e^{\sigma - 1} \frac{L}{2} \qquad e = \left[\sum_j p_j^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

$p_i X_{ij}^d$ is the revenue received by the exporter and X_{ij}^d/t are the number of units arriving in the importing country

The price per unit in the importing country must be $p_i t$ ($p_i X_{ij}^d = (p_i t) X_{ij}^d / t$).

$$e_i = \left[N_i^d p_i^{1-\sigma} + N_j^d (p_j t)^{1-\sigma} + N_i^h p_i^{1-\sigma} + N_j^h p_i^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

where the superscript d denotes domestic or national firm and m denotes a two-plant horizontal multinational, and subscripts i and j denote the two countries.

Consider *identical countries*.

Assuming that marginal cost of production is the same for both domestic and multinational firms,

the pricing equation in the model says that all varieties will have the same (domestic) prices in equilibrium.

Assuming that the relevant firm types are active in equilibrium, the demand functions for the various X varieties sold in country i are:

$$X_{ii}^{d} = X_{ii}^{h} = X_{ji}^{h} = p_i^{-\sigma} e_i^{\sigma - 1} L/2 \qquad X_{ji}^{d}/t = (p_j t)^{-\sigma} e_i^{\sigma - 1} L/2$$

where the second equation can also be written as:

$$X_{ji}^{d} = p_j^{-\sigma} t^{1-\sigma} e_i^{\sigma - 1} L/2 \qquad X_{ji}^{d} = p_j^{-\sigma} \phi e_i^{\sigma - 1} L/2$$

where ϕ is the "phi-ness" of trade: ϕ = 1 (t = 1) is free trade, ϕ = 0 (t = +inf) is autarky.

Zero profit conditions for d and m firms located in country i are markup revenues equal fixed costs:

$$(p_i/\sigma)X_{ii}^d + (p_i/\sigma)X_{ij}^d \leq fc_x^d$$

$$(p_i/\sigma)X_{ii}^m + (p_i/\sigma)X_{ij}^m \leq fc_x^m = \beta fc_x^d$$

Using the demand functions for X_{ii} and X_{ij} above, these are:

$$p_i^{1-\sigma} e_i^{\sigma-1} L/2 + p_i^{1-\sigma} t^{1-\sigma} e_j^{\sigma-1} L/2 \leq \sigma fc_x^d$$

$$p_i^{1-\sigma} e_i^{\sigma-1} L/2 + p_i^{1-\sigma} e_j^{\sigma-1} L/2 \leq \sigma fc_x^m = \beta fc_x^d$$

Suppose that we pick values of parameters such that national and multinational firms can both just break even in the two identical countries.

Then the ratio of the two zero-profit conditions give us the critical relationship between trade costs and fixed costs for indifference.

$$\frac{(1 + t^{1-\sigma})}{2} = \frac{fc_x^d}{\beta fc_x^d} \qquad 2 > (1 + t^{1-\sigma}) = \frac{2}{\beta} > 1$$

$$1 + \phi = 2/\beta$$

Indifference between national and multinational firms

Higher trade costs allow for lower firm-level scale economies (higher β) for firms to be indifferent as to type.

Freer trade (larger ϕ) require a higher level of multi-plant economies of scale (knowledge non-rivaledness or jointness) to suppose multinationals.

No multinationals in free trade unless added cost of a second plant is zero ($\beta = 1$).