# Getting Started with General Equilibrium

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# Overview of the Talk

1. Alan S. Manne
2. The Tale of the Elephant and the Rabbit
3. General Equilibrium
4. A Model of Porters and Mules

# Alan S. Manne

- Born 1925 in New York City
- Received his AB from Harvard in 1943 (18 years old)
- Served as an officer on a destroyer in the Pacific during WW II. Known by the sailors as "Plato".
- Ph.D. in economics in 1950 from Harvard University.
- He produced a pioneering *application of linear programming to oil refinery operation*.

# Alan S. Manne (cont.)

- Faculty appointment in economics at Yale.
- Mentored by Tjalling Koopmans.
- Research focused on multisectoral models for development planning, economic equilibrium computation and analysis, energy-economic interactions and greenhouse gas mitigation policy.
- Ford Foundation in India, Rand Corporation, and several extended visits to IIASA.
- Joined the Stanford Operations Research department in 1967.

# Stanford OR Department in 1967

From Left: Dantzig, Manne, Hillier, Iglehart, Veinott, Kalman, Lieberman, Arrow, Cottle

# My Road to Stanford OR

# Walkway Assembly in Nepal, 1978

# Syange Bridge Inauguration

# Topic of my first meeting with Alan Manne, September 1980

It was six men of Indostan
To learning much inclined,
Who went to see the Elephant
(Though all of them were blind),
That each by observation
Might satisfy his mind

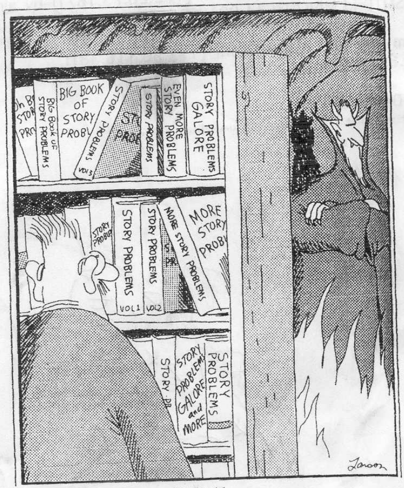
The First approached the Elephant,
And happening to fall
Against his broad and sturdy side,
At once began to bawl:
"God bless me! but the Elephant
Is very like a wall!"

The Second, feeling of the tusk,
Cried, "Ho! what have we here
So very round and smooth and sharp?
To me 'tis mighty clear
This wonder of an Elephant
Is very like a spear!"

The Third approached the animal,
And happening to take
The squirming trunk within his hands,
Thus boldly up and spake:
"I see," quoth he, "the Elephant
Is very like a snake!"

The Fourth reached out his eager hand,
And felt about the knee.
"What most this wondrous beast is like
Is mighty plain," quoth he;
"'Tis clear enough the Elephant
Is very like a tree!"

The Fifth, who chanced to touch the ear,
Said: "E'en the blindest man
Can tell what this resembles most;
Deny the fact who can,
This marvel of an Elephant
Is very like a fan!"




Hell's library

**Modeling Energy-Economy Interactions: Five Approaches**, edited by Charles Hitch. Published by Resources for the Future

"Energy-Economy Interactions: The Fable of the Elephant and the Rabbit?" by William Hogan and Alan S. Manne.

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1977

# Motivation

- In many energy policy studies, the energy sector is appropriately viewed in isolation from the remainder of the economy.
- In some situations this may be inappropriate, as there may be *two way interdependence* between energy markets and the rest of the economy.
- Even a large change in energy markets may represent a small fraction of aggregate economic output.
- There may be virtual one-way linkages: growth in aggregate GDP influence energy demand, but not vice versa.
- If, however, two-way linkages are important, then the analysis of energy market issues demands an economy-wide perspective.

# Objective

- Before embarking on a complex analysis of the interdependence effects (of energy and ecoonomic activity), it is useful to make a rough assessment of their magnitude.
- Here we explore a simple *model* for organizing concepts and parameters.
- This aggregate model provides insights and a perspective on the range of energy policy impacts on the remainder of the economy.

# The Elephant and the Rabbit

- The energy value share of GDP is typically on the order of 4-5% in industrial countries.
- This is something like *elephant-rabbit stew*. If such a recipe contains just one rabbit (the energy sector) and one elephant (the rest of the economy), doesn't it still taste very much like elephant stew?
- But what if energy prices double, triple or quadruple, and there is sufficient time for the economy to respond? How much will this cost the rest of the economy?
- For large reductions in energy use, the value share of energy in aggregate output need not remain fixed. If the value share rises, the metaphor of the elephant and the rabbit may no longer be appropriate.

# Substitution

- Production processes are not fixed immutably. Insulation, energy efficiency improvements and "input juggling" in production processes can all alter the energy requirements for a given level of output.
- *Flexibility* in energy utilization is the next essential element after the energy value share in measuring the magnitude of energy-economy feedback.
- Economists describe the responsiveness of technology by the *elasticity of substitution*.
- There are significant differences between *long-run* and *short-run* elasticities. Here we focus on the former.

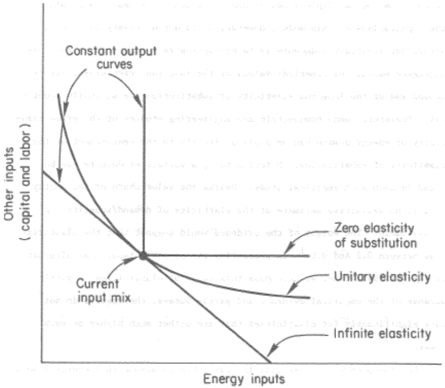



FIGURE 1. THE ELASTICITY OF SUBSTITUTION CONCEPT

# Empirics

- If we focus on models with a constant elasticity of substitution (CES), we can build a coherent, self-consistent model of energy-economy interactions.
- Most empirical estimates of the elasticity value converge on values between 0.2 and 0.6.
- It is a reasonable assumption in many economies to assume constant energy intensity in the absence of policy measures: if the price of energy remains unchanged, energy demand grows proportionally to GDP.

# Policy Context *in 1977*

- Multiple motivations for policy measures leading to energy conservation: environmental protection, national security and *sustainability*.
- Feedback issues are central when we consider two questions:
  (i) What size of tax is required to achieve a target reduction in energy use? and
  (ii) What is the resulting impact on GDP?
- Formally, focus on a future economy (2010) in which the steady-state forecast energy demand is 220 quadrillion BTU.
- Contemplate policy measures which lead to a substantial reduction in energy demand

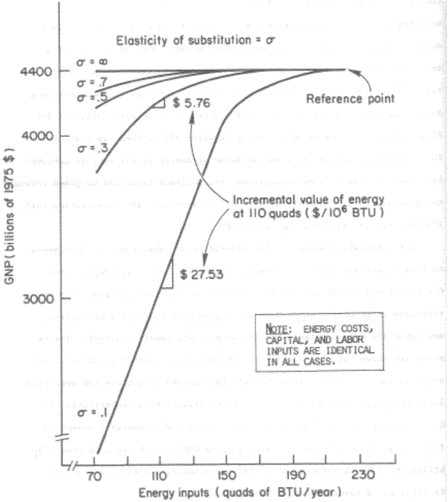



FIGURE 2. ECONOMIC IMPACTS OF ENERGY REDUCTIONS IN THE YEAR 2010

# Capital Mobility

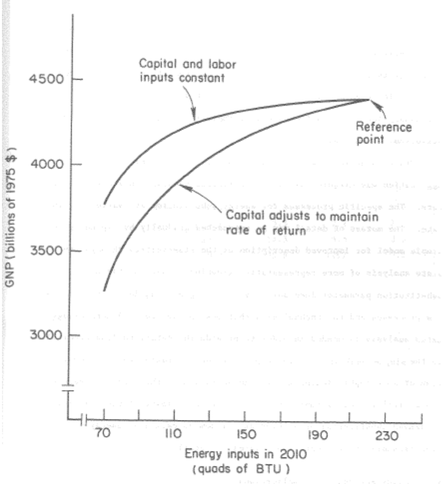



FIGURE 3. ECONOMIC IMPACT OF ENERGY SCARCITY IN THE YEAR 2010 FOR ALTERNATE CAPITAL ASSUMPTIONS (ELASTICITY OF SUBSTITUTIONS $\sigma = 0.3$)

# The Model

Zero profit:

$$Y = P_E E + P_R R$$

where

$E$ represents energy demand

$R$ represents inputs of labor and capital

$Y$ represents aggregate output

Technology:

$$Y = F(E, R)$$

where $F()$ is positive, differentiable and convex function exhibiting constant returns to scale.

# Optimization

Firms are assumed to operate as though they solve:

$$\max F(E, R) - p_E E - p_R R$$

hence, we have:

$$\frac{\partial F}{\partial E} = p_E$$

It then follows that the elasticity of output with respect to the input of $E$ is equal to the energy value share:

$$\frac{\partial F}{\partial E}\frac{E}{Y} = \frac{p_E E}{Y}$$

# The Energy Value Share

If the reference value share of energy equals $s$, then

$$\frac{Y}{Y_0} \approx \left(\frac{E}{E_0}\right)^s$$

If $s = 0.04$, then a 50% reduction in $E$ would require a 2.7% reduction in $Y$. If $s = 0.1$, then a 50% reduction in $E$ would require a 6.6% reduction in $Y$.

Problem: *the value share of energy does not remain constant for substantial changes in user costs of energy*.

# Observation

Note that the energy value share is given by:

$$s = \frac{P_E E}{Y} = a^{\sigma}(P_E)^{1-\sigma}$$

When $\sigma$ is less than one, an increase in the energy price leads to an increase in the energy value share.

If aggregate output is approximately independent of $E$ (i.e., when energy is a small fraction of output), then the elasticity of substitution is approximately equal to the price elasticity of energy demand. The exact elasticity is given by:

$$\epsilon = \frac{-\sigma}{1-s}$$

# Policy Implications

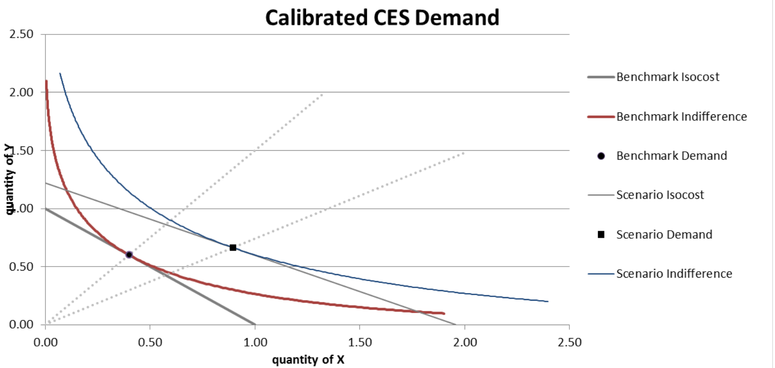
- When technological options are limited, substantial reductions in energy use may be very costly.
- When elasticities of substitution are large, the cost may not be so difficult.
- Implications for energy-economy analysis are clear: when energy demand substitution is low, there is a likely need for two-way interaction between the energy sector and the rest of the economy.
- When demand elasticities are high, a partial equilibrium approach may be perfectly adequate.

# Objections and Caveats

- *Aggregation bias* may hide substantial opportunities for zero-cost energy conservation measures which may exist in individual sectors of the economy
- The aggregate elasticity of substitution does not describe new processes and technologies which may be needed in order to lend credibility to this analysis.
- The *isoquant* employed here summarizes technologies which are "on the shelf". Within this model there is no explicit role for policies which foster innovation and thereby alter the isoquant.
- A large part of the motivation for building larger and more complex models can be viewed as a means of overcoming the shortcomings of such "back of the envelope" calculations.

# Contemporary Relevance

- The thematic and policy dimensions of Manne and Hogan's 1977 paper remain topical 35 years after publication: energy security, environmental cost and sustainability are all at the forefront of the academic research agenda in energy economics.
- What may be less obvious about this paper is its methodological relevance. The CES demand system introduce here continues to be a workhorse of theoretical and applied economic equilibrium analysis.
- The CES function is a key component in multi-sectoral general equilibrium models which exploit the inherent global convexity and local flexibility.




| | | | | | |
|---|---|---|---|---|---|
| x0 0.40 | px0 1.0 | sigma 1.5 | epsilon 2 | px 0.625 | py 1 |
| Benchmark Value Share of X | Benchmark Price of X (py0=1) | Elasticity of Substitution | Price Elasticity of Aggregate Demand | Scenario Price of X | Scenario Price of Y |

# The Arrow Debreu Framework

A general-equilibrium model consists of:

- Profit-maximizing firms.
- Markets, typically with supply and demand mediated through prices.
- Budget-constrained utility-maximizing households.

In policy analysis, numerically *calibrated* versions of these models are referred to as *Computable* or *Applied* General Equilibrium Models (CGE).

N.B. Existence of a substantive CGE calculation should always be viewed as a *necessary* but not a *sufficient* condition for justifying the merits of a particular policy proposal.

# Strengths and Weaknesses

- Key *advantage* of the general equilibrium framework: transparency, logical coherence and consistent accounting of both direct and indirect effects.
- The CGE approach can be consistent with the principal of Occam's Razor: "A scientific theory should be as simple as possible, but no simpler."
- Can be either calibrated or estimated. Hence, it is possible to formulate a model which matches both current economic statistics (supply and demand) and historical evidence about the responsiveness of quantity to price.
- Theoretical coherence provides a means of formulating models which perform better "out of sample".
- Key *disadvantages* of the approach: potential complexity, reliance on optimizing behaviour, data requirements and incompatability with engineering perspective.

# The Fundamental Problem (Cottle)

*Complementarity* is a feature of constrained optimization problems. In an optimal program, Koopmans explains:

- Every process in use makes a zero profit
- No process in the technology makes a positive profit
- Every good used below the limit of its availability has a zero price
- No good has a negative price

Credit for these insights are given to the contributions of Lerner, Samuelson, and Kantorovich.

# Applications of Complementarity in Engineering

I. Kaneko, G. Maier, On optimum design of plastic structures

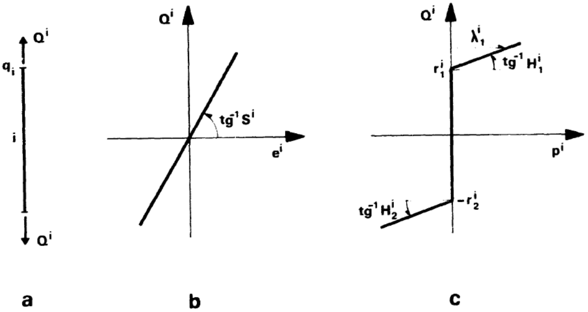



Fig. 2. Elastic–plastic member behaviour.

# Tjalling Koopmans' Nobel Address – 1975

If economics is the study of the "best" use of scarce resources, where does the boundary with management science, engineering, climate science lie?

Optimization and economic theory were in a very exciting period during the 1950s and 60s.

The simplex algorithm for linear programming opened a new frontier for the use of computers in management.

Economists learned to relate ideas from optimization theory to better understand the role of market prices in the allocation of goods and factors of production.

There were high expections (perhaps too high).

Academics were not in full agreement about the appropriate role of economics in academic and policy discourse.

British economist A. C. Pigou (1920) as quoted by Koopmans:

*"...it is not the business of economists to teach woolen manufacturers how to make and sell wool, or brewers how to make and sell beer ...".*

Many European economists, particularly German, Dutch and Scandinavians, disagreed. Models of production planning and economic efficiency were perceived as valuable contributions to both the theory of the firm and public economics.

The Arrow Debreu general-equilibrium model consists of:

- Profit-maximizing firms.
- Markets, typically with supply and demand mediated through prices.
- Budget-constrained utility-maximizing households.

# Firms and Production

*Activities* in the Arrow-Debreu framework transform some goods and factors into others goods. These may include trade activities which transform domestic into foreign goods, activities which transform leisure into labor supply, and more conventional production activities which transfer labor, capital and materials into products.
Activities are most usefully represented by their dual, or cost-functions.
Equilibrium conditions relate marginal cost to the value of output with *complementary slackness* between profit and activity level.

# Markets and Prices

General-equilibrium models consist of market clearing conditions. A *commodity* is a general term that includes goods, factor of production, and even utility.

Market clearing conditions in a general equilibrium model relate supply and demand. Prices exhibit *complementary slackness* with excess supply.

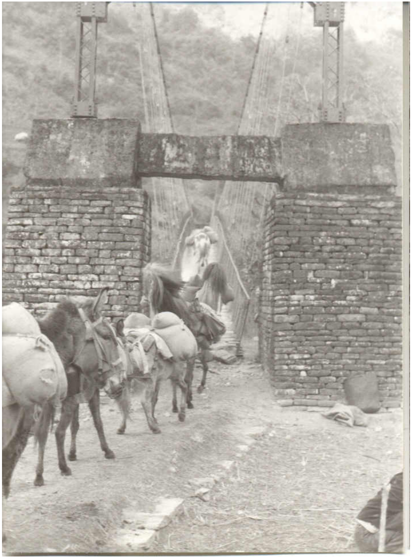
# Consumers (Households and Governments)

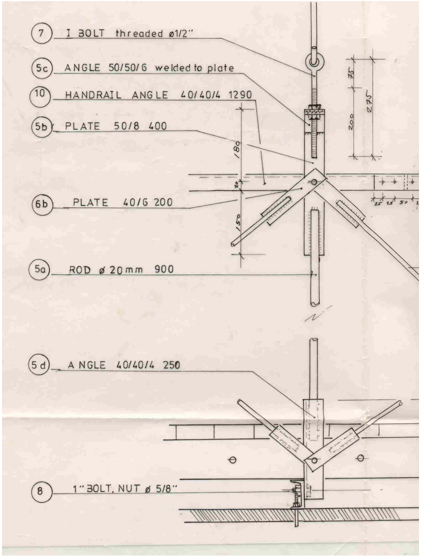
Consumers in the Arrow Debreu framework are endowed with goods (and possibily tax revenue), and they demand commodities. Quantities demanded arise from optimization subject to a budget constraint.

# Returning to Nepal in 1985

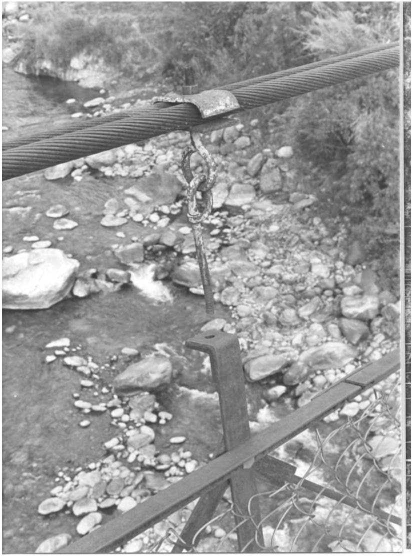
- The price of rice in Manang has falled by 70%
- 200-300 mules on the trail from Dumre to Manang
- Apart from porters working for trekking agencies, almost no porters to be seen on the trail.

# The Bridge at Syange (1985)





7 I BOLT threaded ø1/2"
5c ANGLE 50/50/6 welded to plate
10 HANDRAIL ANGLE 40/40/4 1290
5b PLATE 50/8 400
6b PLATE 40/6 200
5a ROD ø 20mm 900
5d ANGLE 40/40/4 250
8 1" BOLT, NUT ø 5/8"
75
275
200
180
20
150
25
25
50



# An Arrow-Debreu Model of the Market for Porters

- Villages ($r$) are uniformly distributed on a square district.
- Commodities ($g$) are endowed to villages in random amounts.
- Representative consumers in each village are endowed with random quantities of goods and unit allocation of time.
- Cobb-Douglas preferences extend over consumption of goods ($c_i$) and leisure ($\ell$):

$$U(C, \ell) = \ell \prod_g C_g$$

- Portering services are required to deliver goods from one village to neighboring villages.
- The shadow price of portering services differs on all routes depending on differences in commodity endowments and the availability of porters.

- Equilibrium prices clear all markets:

$$\underbrace{\omega_{gr}}_{\text{Initial Endowment}} + \underbrace{\sum_{r'} E_{gr'r}}_{\text{Imports}} = \underbrace{C_{gr}}_{\text{Consumption}} + \underbrace{\sum_{r'} E_{grr'}}_{\text{Exports}}$$

- Individuals allocate their time to leisure and portering:

$$\bar{L} = \underbrace{\ell_r}_{\text{Leisure}} + \underbrace{\sum_{g,r'} X_{rgr'}}_{\text{Portering}}$$

- Budgets are determined by prices and endowments:

$$\underbrace{M_r}_{\text{Income}} = \underbrace{P_r^{\ell}\bar{L}}_{\text{Value of Time}} + \underbrace{\sum_{g} P_{gr}\omega_{gr}}_{\text{Value of Endowment}}$$

- Individual choices are optimizing:

$$C_{gr} = \theta \frac{M_r}{P_{gr}}, \quad \ell_r = \theta \frac{M_r}{P_r^{\ell}}$$

- Arbitrage constraints relate commodity prices, transportation arbitrage constraints to neighboring villages:

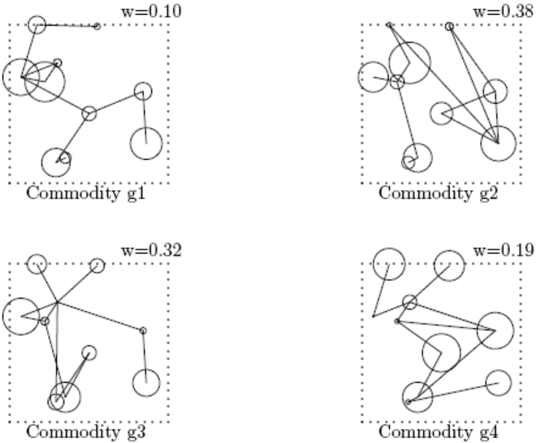
$$\underbrace{P_{gr}}_{\text{Purchase Price}} + \underbrace{PT_{rr'}\phi_{grr'}}_{\text{Transport Cost}} \geq \underbrace{P_{gr'}}_{\text{Sales Price}} \qquad \forall r' \in \mathcal{N}_r$$

- When delivering a load of good $g$ from $r$ to $r'$, the porter returns with no load if there are no goods to be transported on the return. The decision to porter loads thus depends on the shadow value of leisure and the market price of transportation services on neighborhood routes:

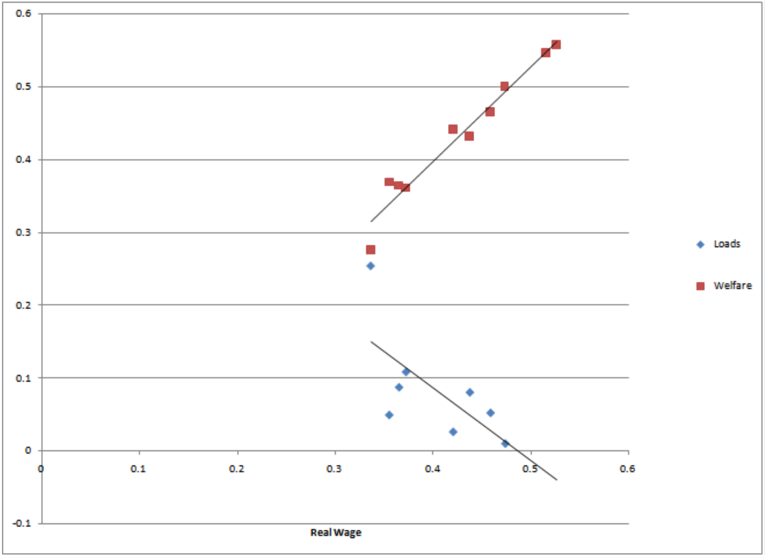
$$\underbrace{P_r^{\ell}}_{\text{Value of Time}} \geq \underbrace{PT_{rr'} + PT_{r'r}}_{\text{Portering Wages}} \quad \forall r' \in \mathcal{N}_r$$

# Porters and Trade

Figure 1: Initial Endowments and Equilibrium Trade Flows



# No Surprise: Poorer People Work as Porters



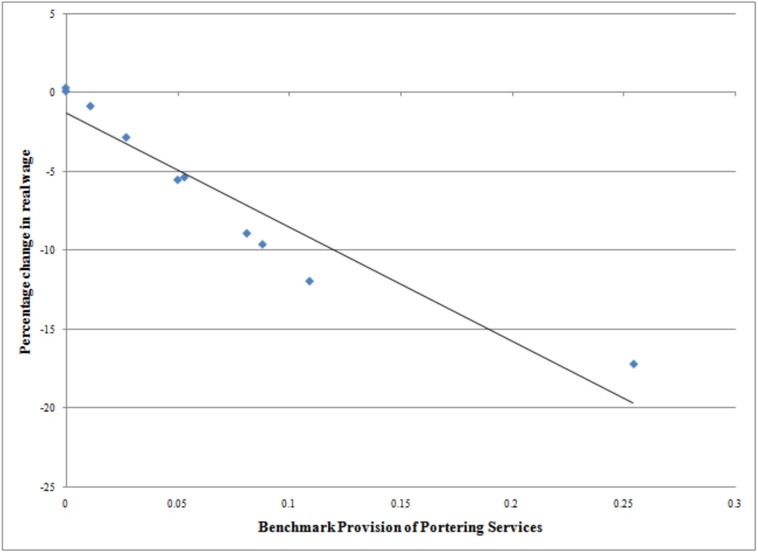
# Introducing Mules

Mules based in region $r$ carry loads in return for compensating payment in goods. The arbitrage conditions for mules operating from region $r$ is:

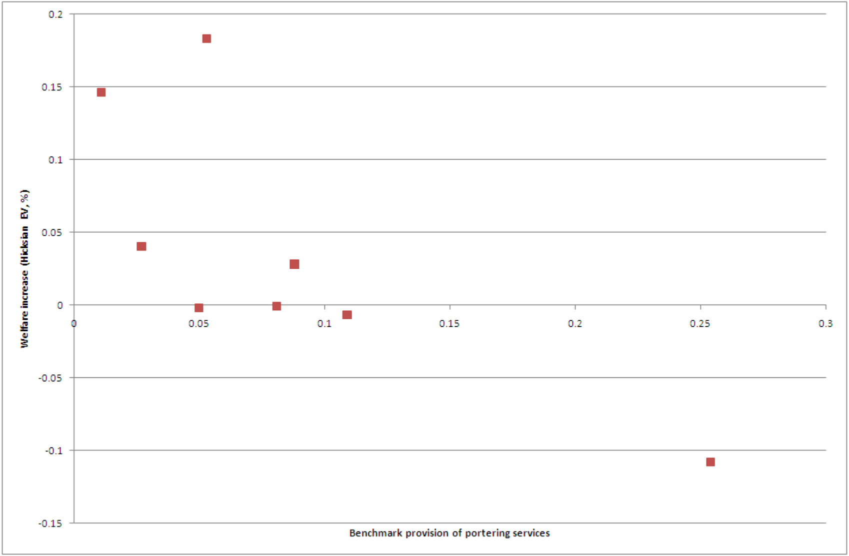
$$\underbrace{\mu \sum_g P_{gr}}_{\text{FeedingCosts}} \geq \underbrace{PT_{rr'} + PT_{r'r}}_{\text{Earnings}} \quad \forall r' \in \mathcal{N}_r$$

When the cost of mules ($\mu$) is sufficiently low, porters are driven from the market and equilibrium wages fall.

# Mules Lower Wages



# Mules Increase Welfare for Most but Not All



# Return to Nepal 2016: Kathmandu Bus Station

# Kathmandu Infrastructure

# Dumre Has Grown

# Water Buffalos are Still Around

# People Still Carrying Rocks

# Horses Still Provide Rapid Transport

# Kids Still Walk Miles to School

# Technology is Evident: Solar Kettles

# Notable Change: Colorful Houses in Lamjung

# Braga monastery is still in operation

## .. and is in excellent condition

# Immaculate Guest Houses in Upper Pissang, Manang

# Women(!) Work as Trail Guides

# Mules Still Cross Syange

# Syange has a New Walkway and the Anchorages are Solid

# Hydro Stations Abound

# Huge Untapped Hydroelectric Potential

# Tourism and Remittances Drive the Local Economy