Homework 2: Project Independence

AAE 526: Quantitative Methods
Fall Semester, 2018

rutherford@aae.wisc.edu

Due 11 October, 2019

Project Independence for Energy Security (PIES) was an initiative announced by U.S. President Richard Nixon on November 7, 1973, in reaction to the OPEC oil embargo and the resulting 1973 oil crisis. Recalling the Manhattan Project, the stated goal of Project Independence was to achieve energy self-sufficiency for the United States by 1980 through a national commitment to energy conservation and development of alternative sources of energy. Nixon declared that American science, technology and industry could free America from dependence on imported oil (energy independence).

For this homework we will implement a stylized PIES model in GAMS and attempt to reproduce results from the paper, "Energy Policy Models for Project Independence" by William Hogan in *Computers and Operations Research* Vol 2, pp 251–271, 1975.

i. Formulate a prototype PIES model as a quadratic program in GAMS which can produce two equilibria, one without constraints associated with capital or steel and another which accounts for these constraints.

ii. Reformulate your PIES model as a linear complementarity program in GAMS and demonstrate that you obtain the same results as in the quadratic program.

iii. Compare your results with those presented in Tables 8 and 9 of Hogan's paper. Can you explain the discrepency?

Comput. & Ops Res., Vol. 2, pp. 251–271. Pergamon Press, 1975. Printed in Great Britain

ENERGY POLICY MODELS FOR PROJECT INDEPENDENCE*

WILLIAM W. HOGAN†
Federal energy administration, Office of the Deputy Assistant Administrator for Data and Analysis, U.S.A.

Scope and purpose—Project Independence was initiated by the President in March of 1974 after the oil embargo, to evaluate U.S. energy problems and to provide a framework for developing a national energy policy. The FEA led the effort which involved over 500 professionals, and resulted in an extensive series of reports in late 1974 and early 1975. Among other outputs, there were predictions of the relationship among a number of key variables such as oil price, demand for oil and other fuels, possible government strategies and gross national product. This article describes the computer program and operations research techniques which produced these predictions, which were the core of the Project Independence analysis.

Abstract—The Project Independence Evaluation System for the quantitative analysis of the national energy policy is outlined. The conceptual framework for determining an equilibrium balance of energy supply and demand is characterized. The computational procedure is indicated and an example is presented. The policy application experience and the computational implications are summarized.

1. INTRODUCTION

Project Independence is the generic title describing the activities organized through the Federal Energy Administration in the continuing development and implementation of a national energy policy for the United States. The analysis supporting these tasks involves many diverse groups and varying viewpoints that must be coordinated and evaluated as part of the policy development. The essential quantitative analysis must incorporate a wide range of judgmental, engineering, and economic information in a manner directed at the careful and explicit consideration of the numerous policy alternatives.

This quantitative analysis is accomplished through the Project Independence Evaluation System. The system generates planning estimates depicting possible states of the energy system with explicit recognition of the effect of relative prices, the potential for fuel substitution, and the technological constraints which inhibit the increase of energy supply. The purpose of the present paper is to outline the structure of the series of judgmental and quantitative models which constitute the Evaluation System with particular attention to the algorithmic procedures and the methods for combining econometric, engineering, and optimization models. Section 2 of this paper summarizes the basic objectives of the Evaluation System. Section 3 describes the conceptual framework for the analytical problem. Section 4 presents a simple example of the energy balance problem addressed. Section 5 summarizes the computational experience judged to be of primary interest. Section 6 outlines the capabilities and uses for policy applications. Two appendices elaborate the central computational procedure and the details of the example problem.

2. OBJECTIVES OF PROJECT INDEPENDENCE EVALUATION SYSTEM

The national energy system is highly interdependent. A variety of energy forms can be employed to satisfy many ultimate requirements, alternative production modes compete for many of the same resources, physical and technological limitations can restrict the supply or potential uses of energy, environmental effects or other externalities can alter the permissable production and use patterns, and new technologies can rapidly transform the available options.

*The views expressed in this paper are solely those of the author and do not represent the position of the Federal Energy Administration.

†Dr. Hogan is the Deputy Assistant Administrator for Data and Analysis of the U.S. Federal Energy Administration. He holds the B.S. degree from the Air Force Academy in Colorado, and the MBA and Ph.D. from UCLA, all in Operations Research. His principal interest is in large scale optimization theory and its applications to energy forecasting. He has published in *Management Science, Journal of Mathematical Programming, Journal of Operations Research, SIAM Review, Econometrica* and the *Journal of Financial and Quantitative Analysis.*

Pervasively, relative prices guide and determine the options that are selected from the many alternatives.

The development of a national energy policy must include a description and assessment of these and other complexities. The evaluation of alternatives must be done within the framework of a flexible system able to include the diverse approaches that are necessary to describe and quantify the many components of the energy system. To perform this analysis, the FEA quantitative analysis system considered several objectives in analyzing alternative strategies.

2.1. *Price sensitivity*

The experience of the oil embargo underscored the importance of recognizing the impact of relative prices. The price system is the primary mechanism for developing an efficient allocation of any economic good, including energy. Changes in prices can affect the demand and supply for energy and the resulting structure of the energy system. The effects of prices must be estimated, quantified, and included directly.

2.2. *Fuel competition*

The competition between fuels is closely related to the issue of price sensitivity. An evaluation of the future must give careful consideration to the possibilities or requirements for the substitution of one energy source for another in response to economic conditions or changes in technical limitations.

2.3. *Technology*

The production and conversion technologies exhibit great variation within the energy system. Geological limitations, conversion efficiencies, fuel requirements, product qualities and other characteristics are useful parameters for describing current or proposed technologies. These should be recognized and included in the evaluations.

2.4. *Resource limitations*

Physical capacities or other resource limitations can inhibit the increase in the potential energy supply. Transportation capacities, refining limits, manpower shortages, or equipment limitations are a few candidates from the lengthy list of potential bottlenecks that must be addressed.

2.5. *Externalities*

By-products or side effects of energy production and consumption influence the assessment of any policy. Environmental impacts are a prime example, and the system must be capable of describing these effects within the context of our present knowledge.

2.6. *Economic impact*

A primary externality of the energy system is the interaction with the remainder of the economy. The Evaluation System must provide descriptions of the economic effect of various energy policies as well as the effect of the economic environment on energy decisions.

2.7. *Regional variations*

Energy production and consumption are not evenly distributed geographically. The distribution system is carefully designed and has little excess capacity. Regional variations must be identified and considered within the Evaluation System.

2.8. *Dynamics*

Lead times, capacity in previous periods, and other time dependent conditions will determine the feasibility of many of the major options. Any assessment should consider the impacts of the dynamics of the evolution of the energy system.

2.9. *Modularity*

The analysis of the energy system is a continuing process which should improve the quality and breadth of understanding. The Evaluation System must support and enhance such accumulation of knowledge. This implies a modular system to permit the expansion of major components or the introduction of new elements as needed.

2.10. *Judgment*

The assessment of policies for the development of the energy system requires expert judgment. The lack of data or the complexity of many of the problems preclude a procedure that does not employ approximations and informed estimates prepared with limited empirical evidence. The Evaluation System must be capable of incorporating such information and of displaying the relative sensitivity of these judgments.

The remainder of this paper is devoted to an outline of the models which address these objectives. The computational procedure is flexible and the results can be replicated. The system meets or is capable of addressing the objectives established above, but with a varying degree of success. The major uses and limitations of the Evaluation System are discussed in section 6.

3. CONCEPTUAL FRAMEWORK FOR THE ENERGY SYSTEM

The energy system is depicted in Fig. 1 as a network wherein production, processing, conversion, distribution, transportation and consumption activities take place.* The prices and capacities for these activities are described in a manner consistent with the dual objectives of preserving price sensitivity and providing explicit recognition of potential constraints on the system. The structure of this framework can be developed by separating the supply and demand sectors.

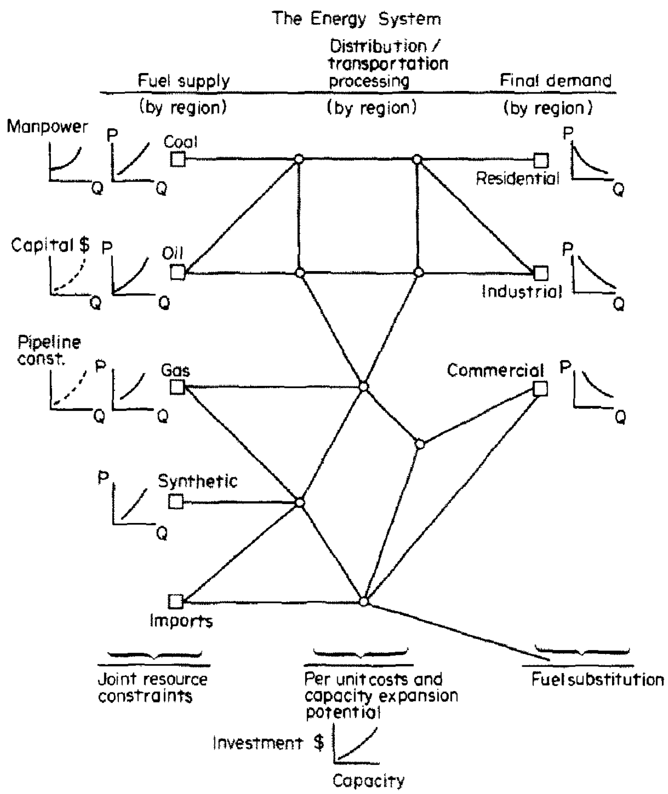

Fig. 1. The energy system.

3.1. *The supply of energy*

The algebraic description of the supply system is presented in Appendix 1. Production, processing and conversion activities are represented as nodes within the energy network and these nodes are connected with links depicting the transportation and distribution system. These

*The term network is not precise. The system does not satisfy any of the conventional formalities of a network or a generalized network. Although the terminology of a network description is employed, the specialized network computational procedures are not applicable.

links permit flexibility in the specification of the geographical regions for the different energy activities.

Potential activities in the energy production system are described by a set of supply curves that identify the prices that must be paid and the non-energy resources that will be consumed at each possible level of operation. Referring to Fig. 1, the regional node for coal is illustrated with two supply curves, one associated with price and one identifying manpower. Therefore, as the production of coal from that region expands the price that must be paid for that coal increases. This increase in price is balanced with the prices of other energy sources as the market choices are simulated. The manpower curve provides similar information for a critical resource that is required for the production of coal. The manpower inputs for coal production are compared with the manpower requirements for other key energy activities and balanced within the limitations of the total manpower availability. Other inputs to the production process (steel, drilling rigs, etc.) can be introduced in an identical manner. This latter feature permits the explicit recognition of scarce resource constraints and the examination of the effects of changing restrictions on key inputs to the production of energy.

The important physical or technological limitations which affect the production of energy are described within the transportation and distribution network. The capacities of the energy transportation systems are represented as upper limits on the total shipments between regions. In addition, the expansion of these capacities can be included as activities governing energy production and the resources required for this expansion will be compared with the other alternatives available in developing the energy system.

The refining and conversion sectors are included as the intermediate nodes of the network, each with a description of its capacities and conversion technologies. As with the transportation system, the operation or expansion of these facilities require various resource inputs and these will be balanced with other requirements for the same materials within the energy sector. The technologies describe the conversion of one energy product into others and, in many cases, identify some of the direct competition between primary energy sources.

The refining and conversion activities are joined with the demand or consumption sectors through an additional set of transportation and distribution links. As before, these links are subject to capacity restrictions which can be modified if sufficient key resources are available when compared with alternative uses in the production or distribution of energy.

The supply curves, conversion technologies, transportation possibilities, costs and resource requirements are produced by supply submodels of the Evaluation System and linked within this framework.

3.2. *The demand for energy*

The estimates of the demand for energy are produced by a demand model. The demand for energy products takes place in different geographical regions and varies with energy prices. Referring to Fig. 1 again, each demand region and sector is depicted with a conventional demand curve. The higher the price of a product, the lower the corresponding demand for that product.

The choice of the activities to be described as demand is somewhat arbitrary, but can be thought of as the final demands for energy. For example, this would include the residential demand for distillate oil in a region but would not include the coal required by electric utilities. This latter "demand" for coal is actually derived in the process of satisfying the final demand for electricity using the competing technologies and primary fuels for the production of electric power.

The simple demand curves of Fig. 1 do not indicate that changes in the price of competing fuels can shift the demand for any fuel. However, this fuel substitution is simulated by the demand model through the empirical development of the relationships between demands and relative prices.

3.3. *Balancing of supply and demand*

Given the prices, resource requirements, and capacity constraints, an integrating model constructs a feasible set of energy flows that satisfies the final demands for energy. The energy supply activities and the demand prices are adjusted during this market simulation to obtain a balanced solution which is in equilibrium. This equilibrium balance is a point where no consuming sector would be willing to pay more for an additional unit of any energy product and no supplier would provide an additional unit of any energy product for less than the prevailing market price.

The implementation of the market simulation is achieved by separating the supply and demand components, approximating the supply side and then adjusting the prices and demands until the system achieves an equilibrium balance. The simulation of the supply side is achieved by assuming that for a given set of prices, demands, and resource capacities, the energy supply system will operate to satisfy demand in a least cost manner. This cost minimization problem can be approximated by a linear program using techniques that have served as the foundation of important previous energy studies.*

For an arbitrary selection of prices and demands, the least cost balancing solution may not be an equilibrium solution; there is no means of guaranteeing that an arbitrary price for estimating product demand will be equal to the price at which the product is supplied. However, the necessary adjustments in the prices are identified and these adjustments are repeated until the equilibrium solution is obtained.

3.4. *Organization of the model*

The supply, demand, and equilibrium balancing components describing the energy system are combined with models of the economy, assessments of non-energy resource availability, and report writers that evaluate energy solutions in terms of the environmental, economic or resource impacts. The relationship of these models is indicated in Fig. 2. An interesting feature of this system is the extensive interface between models of fundamentally different character combined within the framework of the network description of the energy sector and the search for a partial equilibrium solution. Econometric, simulation, accounting, and optimization models are included in this system, each exploiting special capabilities for the relevant components of the problem.

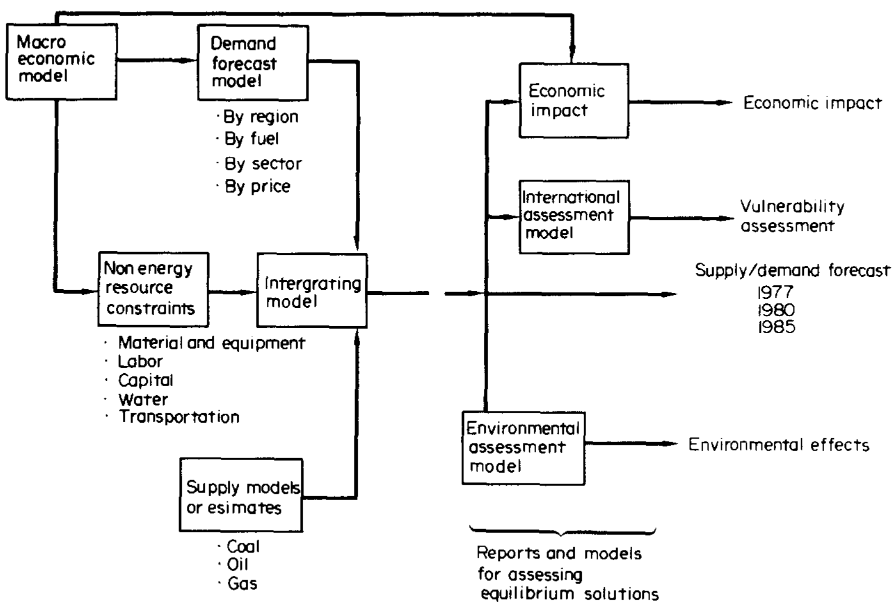

Fig. 2. The FEA project independence evaluation system.

The models indicated by the major blocks in Fig. 2 interface via data arrays but are not connected in real time. The communication via arrays simplifies the exchange of information and maintains the modular characteristic of the system. In addition, necessity dictated a simple interface if the complex models were to be combined in the short time available.† Several of the components have a long history of development and are either machine dependent or require

*The linear programming description of the supply system is conceptually equivalent to the models supporting the Emergency Energy Capacity Study of the Office of Emergency Preparedness or the Energy Hydrocarbons study of the PACE Company of Houston, Texas [5]. The model for the former study has been modified to assemble the supply component for the Project Independence Integrating Model.

†The effort was initiated in April 1974. The *Project Independence Report* was published 7 November 1974.

specialized software. The complete system, as originally developed and currently used, employs several different languages and computer systems. Therefore, the construction of the system required either extensive reprogramming or a simple procedure for model interface.

The model of macroeconomic activity is a well known system developed by Data Resources, Inc. [4] and is one of several major econometric models of the United States economy. Although this behavioral model includes over 1,000 equations relating more than 1,200 variables, relatively few of these variables are used directly. This model is run infrequently under appropriate assumptions to produce forecasts of economic activity in terms of GNP, production indices, rate of inflation, and related parameters. These outputs are used to drive the demand model. However, there is no direct feedback to this macroeconomic model and alternate models or forecasts can be substituted routinely.

The resource constraint elements consist of a large data base which records the coefficients of demand for non-energy resources for the alternate energy activities included in the system. These coefficients are employed to construct constraints for the equilibrium solution if capacities are known or to prepare ex post summaries for off-line evaluation of potential bottlenecks.

The supply model component consists of a variety of procedures used to construct stepwise approximations for the energy supply curves. These range from non-automated engineering analysis in the case of coal to the complex software of the National Petroleum Council for the estimation of the supply of oil and gas [8]. These have been constructed, for coal, or modified, for oil and gas, to produce annual data arrays that provide a regional description of potential supply as a function of price. These tables of prices and associated quantities are used in the Integrating Model to build the supply curve approximations that will be employed in the equilibrium calculations.

In contrast to the engineering analysis and process oriented descriptions of the supply models, the demand model is a behavioral econometric model. The supply models are constructed using detailed technical knowledge, resource estimates, cost and profit calculations and judgment about the economic actions of the industry. The process of demand and substitution of energy goods is more complex. To capture this complexity, the demand for energy is not represented in terms of its final use as energy but as the demand for the variety of energy products in a the using sectors. The structure of demand and substitution is postulated in terms of relative prices and the parameters of these relationships are estimated using econometric techniques. The resulting system consists of over 800 behavioral relationships governing the demand for energy in 40 product and sectoral combinations. The demand model has a lag structure which approximates the dynamics of price responses but could complicate the interface with the description of supply and distribution. This complexity is circumvented by a simple approximation. Essentially, the demand model is simulated with a price path to produce quantities which are interpreted as a point on the appropriate demand curve. The full demand model is then iteratively differentiated and the resulting derivatives are used to construct a demand curve through the estimated point. The data array of this simplified picture is transferred to the integrating model as described in more detail in Appendix 1.

These demand, supply, and resource assessment models are combined through the series of programs which consltitute the integrating model. This is the software which constructs and solves the network description of the energy system to obtain a partial equilibrium, balancing prices and quantities for all energy products. The supply, conversion, distribution and resource limits of the network are prepared in the context of a linear program using PDS, a commercially available matrix generation language [9]. The APEX linear programming code [1] is used to solve the supply problem for given demands and prices and to control the iterative price adjustment procedure in the search for an equilibrium solution. This algorithm is developed in Appendix 1.

The quantity flows and prices of the equilibrium energy sector solution provide the input to a series of evaluation or report writing programs that relate the solution to particular problems under consideration. These reports, over 20 in total, range from an executive summary of the energy balance to detailed classification and compilation of associated environmental residuals, water requirements or implied non-energy resource usage for those inputs which have not been considered directly in the energy system network. These reports are written in the PDS report writing language and are an essential link for converting the gross outputs into usable information.

The implementation of this procedure is described in more detail in Appendix 1. The model needed to portray the entire energy system is large and complex. Many regions, fuel types, conversion technologies and transportation alternatives are combined with a highly disaggregated representation of the relation between prices and demands. Consider, for example, the case of oil. The production of conventional crude oil is described for fourteen regions at any of more than a dozen price and quantity levels and includes three major by-products that also contribute to the supply system. Once oil is produced, it may move by feasible pipeline, barge or tanker routes to any of the seven refining centers. When combined with other inputs, several refining technologies or yields must be selected in each refining center to determine the transformation of the crude oil into varying combinations of four types of refined products. These products are shipped in turn by varying modes of transportation to the nine demand centers for final consumption or conversion into electric power. At each stage, capacities on processing, transportation, and key resource inputs are recognized. The level of activity of each variable must be determined simultaneously with the evaluation of all other fuel types and energy demands.

The concepts of the quantitative analysis can be illustrated with an example. A streamlined problem of this type is presented and discussed in section 4.

4. ILLUSTRATIVE EXAMPLE

This section structures an illustrative problem for the energy system requiring the supply of two forms of primary energy, oil and coal. Coal is mined and shipped to the demand regions for direction consumption. Crude oil is pumped and transported to refineries where it is converted into two products, light and heavy oils. These products are shipped in turn to the demand regions for final consumption. For simplicity, only two production or consumption regions are identified for each stage.

A number of data tables are presented to describe the potential and price for the production (Table 1) and transportation (Table 2) of coal, the production (Table 3) and transportation (Table 4) of crude oil, the technology of the refineries (Table 5), the costs of transporting oil products (Table 6), and the price effects on demand (Table 7).

A number of individual capacity restrictions are included. In addition, the important resource constraints will be illustrated by the use of joint limitations on new available capital and steel for input in the productive processes.

An important approximation that is included in the linear programming formulation is the procedure for estimating and representing the variety of supply curves. In most cases, the direct construction of the supply curves is not yet possible and engineering estimates or other expert judgments must be substituted. This data collection problem is one of the largest and most important efforts within the Project Independence analysis. To make this effort feasible, each curve is actually represented by a step function that approximates the underlying supply curve at a number of points. This reduces the data collection problem to identifying several levels of production and permits the direct representation of the supply curve within the basic linear program.

For the example problem, these data are contained in Table 1.

Table 1. Resource requirements for production levels

	Production (ton/D)	Minimum price/ton($)	New capital/ton	Steel/ton
	0–300	5	1	1
Coal	300–600	6	5	2
region 1	600–1000	8	10	3
	0–200	4	1	1
Coal	200–500	5	5	4
region 2	500–1000	7	6	5

Coal is mined in two regions and is considered to be of uniform quality. Three levels of production have been specified to identify three points each on the underlying supply curves for price, new capital, and steel. Figure 3 illustrates the approximation to the curve for steel requirements to produce coal in region 1.

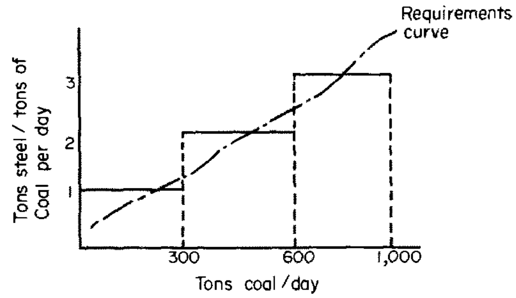

Fig. 3. Steel requirement coal region 1.

After extraction, the coal must be shipped from the mines to the consuming centers. The costs of this transportation are listed in Table 2.

Table 2. Transport costs ($/ton)

	Demand center 1	Demand center 2
Coal region 1	$1·00	$2·50
Coal region 2	0·75	2·75

As with coal, oil can be produced at differing output levels for differing levels of price. It is assumed that there are two potential levels of oil production corresponding to two points on the supply curve. The price and resource requirements for these production levels are shown in Table 3.

Table 3. Resource requirements for production levels

	Production level (MB/D)	Minimum price/barrel	New capital/barrel	Steel/barrel
Oil	0–1100	$1·00	0	0
region 1	1100–2300	1·50	10	4
Oil	0–1300	1·25	0	0
region 2	1300–2400	1·50	15	2

The cost of shipping oil from producing regions to refinery sectors are:

Table 4. Transport costs ($/barrel)

Oil region 1	$2·00	$3·00
Oil region 2	4·00	2·00

The technologies of the refining sectors are represented by showing a range of possible yield patterns. The refineries differ in their normal operating mode in terms of the yields for each product. The percentage yeild splits for the two refinery centers are summarized in Table 5.

Table 5. Refinery yields and cost

	Refinery 1	Refinery 2
Light oil	0·60	0·50
Heavy oil	0·40	0·50
Cost/barrel	$6·50	$5·00

After refining, the products must be shipped from the refineries to the demand centers. The costs of such transport are given in Table 6.

Table 6. Transport costs ($/barrel)

	Demand center 1	Demand center 2
Refinery 1	$1·00	$1·20
Refinery 2	1·00	1·50

This production, conversion and distribution system must satisfy the demands for light oil, heavy oil, and coal. Throughout this analysis these demands are assumed to be price sensitive and this is characterized by introducing elasticities. (The elasticity of demand of one product with respect to the price of another is the ratio of the percentage change in demand to the percentage change in price.) For purposes of the example, the elasticities of Table 7 are employed.

Table 7. Elasticities

	Light	Heavy	Coal
Light ($16,1200)*	−0·5	0·2	0·1
Heavy ($12,1000)	0·1	−0·5	0·2
Coal ($12,1000)	0·1	0·2	−0·75

*The partial elasticities are evaluated at these prices and quantities.

These example data for capacities, production, prices, and demands are converted into a formal model in Appendix 2.

4.1. *Market equilibrium without constraints*

To illustrate the importance of obtaining the market equilibrium and to highlight the potential impact of the resource constraints, the example of the previous section is balanced without the additional constraints. The next section will examine the effect of imposing these important resource constraints.

Consistent with the assumptions of Table 7, the initial estimates of the equilibrium prices and demands were taken as $16, $12, $12 and 1200 MB/D, 1000 MB/D, 1000 Tons/Day for light oils, heavy oils, and coal, respectively in each of the two regions. The equilibrium balance proved to be different. The final prices and quantities in the demand regions are summarized in Table 8.

Table 8. Price and quantity summary

	Price		Quantity	
	Demand region 1	Demand region 2	Demand region 1	Demand region 2
Light oil	$12·5	$12·6	1252	1266
Heavy oil	9·4	9·4	1041	1055
Coal	9·3	11·0	1102	998

Although the entire production capacity of coal was utilized, a portion of the potential oil remained untapped in the most expensive increment of production in producing region 1.

The first refinery handled throughput of 2110 MB/D but the second refinery ran at the higher level of 2504 MB/D reflecting the $1·50 cost differential and different yield patterns.

This short description is supplemented by a detailed balance presented in Appendix 2. It illustrates the possibilities of price adjustments in the demand regions and the fact that regional differences can occur. An important issue that must be addressed is the effect that resource constraints will have on the market solution. In particular, it is important to demonstrate that the procedure can accommodate judgments about the limited availability of key inputs. The next section shows how constraints on steel and new capital availability can be embodied in the procedure and summarizes their impact on these equilibrium solutions.

4.2. *Market equilibrium with constraints*

The solution of the example in the previous section follows a standard economic approach: identify the equilibrium point as the intersection of the supply and demand curves. The advantage of this procedure is that it preserves the recognition of the important price effects. A major criticism of this balance is that the supply curves are not constructed with adequate recognition of physical or engineering constraints and potential limitations on the availability of resources. One of the objectives of the evaluation system is to preserve explicit price sensitivity while accommodating the constraints on the use of key resources. To illustrate the capability to introduce joint resource constraints, an equilibrium balance for the example problem is recomputed with imposed limits on the total use of new capital and steel.

The unconstrained market equilibrium of the previous section required an input of 38,000 + units of new capital and 13,000 + units of new steel. This section assumes supply limitations of 35,000 and 12,000 units of capital and steel respectively. The new equilibrium prices and quantities in the consuming regions are summarized in Table 9 for comparison with the corresponding values in Table 8.

Table 9. Price and quantity summary

	Price		Quantity	
	Demand region 1	Demand region 2	Demand region 1	Demand region 2
Light oil	\$15·4	\$15·6	1205	1229
Heavy oil	11·5	12·0	996	1020
Coal	11·3	13·4	996	910

Inspection of Table 9 indicates a small reduction in the quantity demanded commensurate with the small reduction in the available capital and steel. However, the assumed demand curves are inelastic and this has produced large increases in price. These large increases in price are necessary to create the required reduction in total demands. This illustrates an important point of the basic Integrating Model: *the model does not imply that all energy is actually sold for the costs of production.* Economic rents or higher profits, induced by a scarcity of key resources, continue to exist and are accounted for in determining the market equilibrium.

Other changes can be identified for this new equilibrium solution. Coal production is now below capacity for the most expensive increment. There is also a redistribution of excess capacity in oil production to reflect the differing requirements of critical resources. The detailed solution is contained in Appendix 2.

5. COMPUTATIONAL EXPERIENCE

Several components of the PIES, such as the model of macroeconomic activity, are familiar systems which have been employed elsewhere. The linear programming description of the supply and distribution system is a natural formulation which is used frequently [5, 6]. The APEX code is quite successful for linear programs of this size (sparse matrix, with 700–2000 rows) and solves the problem without an advanced basis in 5 min of CDC 6600 time. While not all the standard components are as successful, e.g., the econometric software is not as efficient as the more structured optimization code, the experience with this system has been predictable with two notable exceptions: the price adjustment algorithm and the problem generation software.

The iterative pricing procedure documented in Appendix 1 is a tatonnement process that was expected to present convergence difficulties. This problem can be viewed as a fixed point computation with 63 variables to be determined. Although research in this area is improving rapidly, the published experience would lead to expectations of 10^4 iterations or greater [11]. The tatonnement used exploits the problem structure to such a degree that the author was expecting reasonable convergence after iterations of order 10^1 or 10^2. Surprisingly, the consistent experience has been successful termination after 6–10 iterations. This has been repeated in hundreds of problems including many cases where the initial estimate is not close to the final solution. The convergence is robust and routine. This success is not fully understood but is reportedly reinforced by the experience with another large energy model which exploits structure in the application of a tatonnment process[3]. Certainly these methods deserve further study and indicate that equilibrium solutions for large models can be approached with heuristic methods with more confidence than expected from the general experience with the more rigorous fixed point theory.

The success of the numerical algorithm was balanced by the difficulties involved in the data handling of the matrix generation and report writing system. Recognizing that it is more difficult to generalize from failure than from success, the implication of this application is that the mathematical software for problem estimation, simulation, and optimization is more advanced than the data handling software for problem description by structuring equations, general matrix generation or report writing. The explanation for this difficulty is not complete. The Integration Model matrix generation saturated the original software and required the extraordinary release of the compiler for the next genreation of language, PDS[9]. Excluding the difficulties associated with debugging the compiler, the successful generation of the initial matrix and the preparation of the first report writer required the intense involvement of the language developers and consumed as much as ten times the computational effort of the solution process. After some tuning, this overhead has been reduced to three to five times the time required for solution but it lacks the robustness of the optimization procedure and is viewed with trepidation when extensive changes are intended.

This last criticism is not intended as a condemnation of matrix generation. In the absolute sense, PDS is a powerful language with the flexibility to deal with many problems and special applications. These models could not have been assembled and interfaced without such a capability and it is used continuously. It is, for example, extremely effective at implementing small revisions in the problem structure. In terms of the overall process of structuring, solving, and analyzing a model, however, the software for structuring and analyzing is more limited than that for solution, in terms of problem size that can be handled. Computer scientists would find this field a rich area for research.

6. CAPABILITIES AND POLICY APPLICATIONS

The framework outlined is flexible and capable of manipulating a large array of quantitative information. The quantification and automation of the system provide the essential capability for the improvement of the large data base and analysis involving several hundred different fuel sources and levels of production. They also permit the replication of the scenarios for modification or critique.

The system is used extensively and is continuously involved in shaping and defining proposed energy policies. The quantitative analysis was pervasive in the preparation of the original Project Independence Report[10] and is used as the primary tool within the Congress as well as the Administration for evaluating alternative energy initiatives. This system is a clear example of successful application of the techniques of quantitative analysis on a large-scale but within the timeframe of decision-making to extend and improve the quality of decisions.

The structure of the system evolved in response to the development of the problem statement and the specification of the policy options considered. Therefore, it is to be expected that the framework described achieves many of the general objectives delineated in section 2.

Price sensitivity, by design, is pervasive in the models. The increasing price of higher energy supplies is described through the detailed construction of component supply curves for many regions, fuel types, conversion technologies and transportation modes. The demand equations employ explicit price elasticities to describe the allocation responses to relative prices. Finally, the definition of the equilibrium balance of the system is stated in terms of a stability condition for relative prices. New production options at higher prices or increasing costs of factor inputs can be accommodated as well.

Fuel competition can be recognized directly or indirectly within the models. The use of cross-elasticities of demand implies that the price of one fuel affects the demand for others, or one fuel is substituted for another. Similarly, the specification of electric utilities can recognize that more than one fuel can be input to produce the ultimate electric output; the choice among fuel types being made during the determination of the equilibrium balance.

The technologies of the various conversion and refining sectors can be summarized through the construction of input-output coefficients or, equivalently, product yield vectors. When combined with capacity constraints within the linear program, a great range of possibilities can be represented and adequate approximations can be constructed.

The chief advantage of the linear programming formulation is that it provides a ready vehicle for including a variety of resource constraints. Any limitation on the system that can be expressed or approximated as a linear constraint can be incorporated directly. The example illustrates this capability for one type of constraint, but it can be exploited in a variety of problems. With few exceptions, the limited data or knowledge available guarantee that any available approximation to a physical or resource limitation will be linear and can be readily appended to the equations. In practice, the capability is used infrequently due to the difficulty of expressing constraints or resource availability for one sector of the economy. Therefore, the resource evaluations are primarily ex post.

Externalities and economic impact measures are developed in the Evaluation System. The system will compute total emission levels or the required amount of steel for energy production, but will not automatically consider the tradeoffs for these items when compared with the rest of the economy or the society. In short, the system balances choices between energy products, but is incomplete in the comparison of energy and other sectors contributing to the general welfare. For these broader analyses, the system may be employed as a parametric tool; e.g., the effects of variations in available steel for energy can be estimated but these must be compared with independent descriptions of the impacts on the remainder of the economy through the use of the macroeconomic components of the overall Evaluation System.

The Evaluation System is designed to handle regional variations. The regional definitions are made independently for each product to correspond to the peculiarities of that fuel. The transportation and distribution system are the links between the varied regional centers. The limitation on the description of regional variation is the availability of data.

The approximations or limitations in the study of dynamics are the most serious deficiencies in the conceptual framework. The determination of an equilibrium balance for a given set of conditions is viewed as a static problem without a time dimension. Obviously, the system changes over time. To accommodate this change, data have been collected to describe the system at different points in time (1977, 1980, and 1985). Increasing supply and demand elasticities are included in the data to reflect the effects of lead times or the delayed response of the energy system. These increasing elasticities, however, do not guarantee that the time path implied by the 1977 equilibrium will be consistent with that of the 1980 equilibrium, for example. This latter problem, when it arises, must be dealt with external to the computational procedure. With some judgment, consistency can be imposed by constructing additional constraints on the equilibrium solution. To some extent, this has been anticipated by the inclusion of data and constraints on the cumulative requirements for key resources. Fortunately, many of the independent components of the system are dynamic and produce projections which are well behaved over time. This regularity has prevented the potential problem from occurring and the equilibrium solutions tend to be consistent over time.

By its very design and through the implementation of the data assembly, the system has dealt with the necessity for including expert judgments. The development of the supply curves, in particular, is accomplished by the expert evaluation and modification of limited empirical data. The ability of the system to append new constraints or generate a variety of sensitivity analysis offers the opportunity for the introduction of new judgments as the evaluation continues. The matrix generator guarantees a high degree of modularity in the system. This process of automation and the basic conceptual framework facilitate the introduction of new sectors or the expansion of existing components. The great bulk of new constraints and activities can be added routinely.

Displaying the supply and demand effects of different import price assumptions is an example of a problem which can be handled directly by the system. Imposing a constraint on refining capacity, electric generation, transportation limits or any of the many activities included in the model will change the relative prices and the supply demand quantities. The effect of such constraints can be readily determined. Shifts in supply curves through accelerated development of resources and shifts in demand curves through accelerated conservation efforts can be approximated and the new equilibrium solutions can be computed. Management of demand or modification of technological efficiencies for the supply of energy will restructure the equilibrium price and quantity relationships and these new solutions can be developed readily. Essentially any policy question that can be stated in terms of changes in supply and demand curves, modifications of energy conversion and distribution technologies, or constraints on the energy supply system can be dealt with in great detail through the quantitative analysis of the system.

The list of policy questions which cannot be addressed using the system alone is equally impressive. Critical review of the limitations of the system and the success in meeting the various objectives can be found in [7]. No reasonable statements can be made about problems depending on a level of detail below the aggregations of the models. This prevents rigorous assessment of local siting problems for new plants, evaluation of the ambient effects of polutants, identification of transportation bottlenecks within regions, or any other problems of an intra-regional nature. Ownership of energy producing and distributing resources is not identified in the model and, therefore, analysis of the possible trends in ownership structures cannot be addressed using the system alone. The interface with the remainder of the world is accomplished through the use of a simple import table and, although conceptually possible, the model does not deal with different world supply and demand patterns.

The list of limitations of the system is not exhaustive but serves to characterize the boundaries of application. Four considerations should determine the applicability of the system to most policy questions: Can the policy question be approximated by a known shift in the supply or demand curves? Can the policy question be approximated with a constraint on the linear program describing supply? Can the policy question be approximated with a change in efficiencies or other input/output coefficients? Can the policy question be accommodated through a simple change in the equilibrium pricing rule? If the answers to all of the questions are negative, then the system is probably not applicable. Otherwise, the system can be used in combination with careful data development and judgmental specification of the necessary changes in the structure.

7. SUMMARY

This paper presents an overview of the quantitative analysis of the Project Independence Evaluation System. The organization of a complex system of engineering, econometric, and optimization models is developed and the central energy system modeling and solution procedures are described. This system represents an example of the successful application of quantitative analysis in the organization and improvement of the decision-making process for a complex problem.

Acknowledgements—The success of this major application of formal analysis in the energy decision process is the result of the combined efforts of hundreds of individuals. Space does not permit the repetition of the names here but most can be found in [10]. The development of improved models and decisions continues unabated by the analysts in FEA and the Federal government.

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APPENDIX 1: CALCULATING AN EQUILIBRIUM BALANCE

A 1.1. *Introduction*

The model suggested in this paper determines a set of energy demands and associated prices such that the supply system can satisfy these demands and no supplier will sell any energy product for less than the prevailing price and no purchaser will pay more than the prevailing price. This appendix develops the computational procedure motivated by this equilibrium concept and describes the broad mathematical structure of the evaluation system.

By usual definition, demand and supply functions determine quantities as a function of prices. Under mild conditions, the inverse functions exist and prices can be viewed as a function of quantities. If Q is a vector of quantities representing various energy products, the equilibrium solution is characterized in terms of the vector of supply and demand price functions (P_s, P_d) as a solution of

$$P_s(Q) = P_d(Q). \tag{1}$$

Under restrictive conditions, the vector functions P are integrable and a new function, T, can be defined as

$$T(\hat{Q}) = f(\hat{Q}) - g(\hat{Q}) \tag{2}$$

where

$$f(\hat{Q}) = \int_0^{Q} P_s(Q) \, . \, dQ$$

and

$$g(\hat{Q}) = \int_0^{Q} P_d(Q) \, . \, dQ.$$

Then any solution to (1) is a stationary point for T. Therefore, if convexity of T applies, a solution to (1) is also a solution to the problem

$$\operatorname*{Min}_{Q} T(Q). \tag{3}$$

This rather heuristic discussion is motivated by the following observations:

With the exception of requirements for the existence of the function g, the problem at hand satisfies the conditions needed to justify (3).

Under strict assumptions about the demand functions, the problem is correctly characterized by (3). Furthermore, the problem in (3) lends itself to a straightforward linear programming approximation which may find a solution to (1).

The solution of the linear approximation of (3) provides an estimate of a solution of (1) and this process may be iterated to search for such a solution.

The difficulty associated with the demand functions centers on the fact that $P_d(Q)$ is not integrable in the problem at hand and g does not exist.*

However, if the cross-elasticities of demand are zero, then little is required to guarantee the existence of g. Therefore, assume for the present that ∇P_d is diagonal.† In this case, g is the sum of the one-dimensional integrals of the component functions of the vector P_d. This fact will be exploited subsequently.

The discussion of section 3 and the example of section 4 outline a linear programming specification of the supply and distribution system. If the vector of activities in the energy system is denoted by X, the price of each activity denoted by C, and the system of equations needed to describe the energy network represented by $A_1X = b$, then the solution of the following problem provides an approximation of f;

$$f(Q) \approx \operatorname*{Min}_{X \in x} CX \tag{4}$$

$$\text{subject to } A_1X = b \tag{4a}$$

$$A_2X = Q \tag{4b}$$

where A_2 provides the transformation of the supply activities that serve to meet the demands Q.

The dual variables, π, associated with the constraints (4b) provide an estimate of the gradient of f or, equivalently, an estimate of $P_s(Q)$.

Continuing to assume the existence of g, (3) is approximated by

$$\operatorname*{Min}_{Q} \left\{ \left\{ \operatorname*{Min}_{X \in x} \begin{array}{c} CX \\ A_1X = b \\ A_2X = Q \end{array} \right\} - g(Q) \right\}. \tag{5}$$

Introducing the perturbation Y, defined as $Y = Q - Q_0$, the problem in (5) becomes

$$\operatorname*{Min}_{Y} \left\{ \left\{ \operatorname*{Min}_{X \in x} \begin{array}{c} CX \\ A_1X = b \\ A_2X = Q_0 + Y \end{array} \right\} - g(Q_0 + Y) \right\} \tag{6}$$

*If g exists then $\nabla g = P_d$ and $\nabla^2 g = \nabla P_d$. In the example and the real problem, P_d is continuously differentiable, implying that $\nabla^2 g$ is symmetric. But ∇P_d is not symmetric.

†This is not necessary but it is convenient in that it retains the most important price effects and is amenable to linear approximation.

or

$$\underset{X,Y}{\mathrm{Min}} \quad \begin{aligned} & CX - g(Q_0 + Y) \\ & A_1X = b \\ & A_2X - Y = Q_0. \end{aligned} \tag{7}$$

Recall that under the assumptions of a diagonal ∇P_d that

$$g(Q) = \sum_{i=1}^{m} \int_0^{Q_i} P_i(Q_i)\,\mathrm{d}Q_i$$

where P_i is the ith component of P_d.

Then

$$g(Q_0 + Y) = \theta + \sum_{i=1}^{m} \int_0^{Yi} P_i(Q_{0i} + Y)\,\mathrm{d}Yi, \tag{8}$$

where

$$\theta = \sum_{i=1}^{m} \int_0^{Q_{0i}} P_i(\hat{Q}_i)\,\mathrm{d}Q_i.$$

Since θ is a constant, it does not affect the solution of (7) and this yields

$$\underset{X,Y}{\mathrm{Min}}\; CX - \sum_{i=1}^{m} \int_0^{Y_i} P_i(Q_{0i} + Yi)\,\mathrm{d}Yi. \tag{9}$$

subject to $A_1X = b$

$$A_2X - Y = Q_0$$

A 1.2. *Approximation with integrable demand functions*

The fact that the diagonal elements of ∇P_d are negative guarantees the convexity of (9) and permits the following approximation.* Introduce a new set of variables $Y_{i,k}$ ($k = -n, -n+1, \ldots -1, 1, 2, \ldots n-1, n$) which will construct a partition of a sufficiently large interval centered at Q_{0i}.

Let $U_{i,k}$ be the upper bound for $Y_{i,k}$ and $Y_{i,-k}$. Hence

$$0 \le Y_{i,k} \le U_{i,k}$$

$$0 \le Y_{i,-k} \le U_{i,k}.$$

Let

$$P_{i,k} = P_i\left(Q_{0i} + \sum_{j=1}^{k} U_{i,j}\right)$$

$$P_{i,-k} = P_i\left(Q_{0i} - \sum_{j=1}^{k} U_{i,j}\right)$$

for $k = 1, 2, \ldots n$. Then, by convexity, for any optimal selection of $Y_{i,k}$ the integrals can be approximated as

$$\int_0^{Y_i} P_i(Q_{0i} + Y_i)\,\mathrm{d}Yi \approx \sum_{k=1}^{n} (P_{i,k}Y_{i,k} - P_{i,-k}Y_{i,-k})$$

and

$$Y_i \approx \sum_{k=1}^{n} (Y_{i,k} - Y_{i,-k}).$$

Using this approximation after dropping constants from the objective function, (9) becomes

$$\underset{X,Y_{i,k}}{\mathrm{Min}}\; CX - \sum_{i=1}^{m} \sum_{k=1}^{n} (P_{i,k}Y_{i,k} - P_{i,-k}Y_{i,-k})$$

subject to $A_1X = b$ (10)

$$A_2X - \sum_{k=1} Y_{i,k} + \sum_{k=1} Y_{i,-k} = Q_0.$$

Hence, any solution of (10) is an approximate solution of (1) with

$$Q_i = Q_{0i} + \sum_{k=1}^{n} (Y_{i,k} - Y_{i,-k}).$$

*The negative ∇P_d is equivalent to the own price elasticities being negative, the standard definition of an economic good.

Furthermore, if

$$0 < Y_{i,k} < U_{i,k}$$

then

$$\pi_i = P_{i,k}$$

The symmetric case holds for $-k$.† Therefore,

$$\pi_i \approx P_i(Q_i), \tag{11}$$

the equilibrium condition defined by (1) and (4).‡

A1.3. *Approximation with the general demand functions*

The heuristic for seeking a solution of (1) is based on a repeated exploitation of (10). In particular, assume that a general demand function has been specified as $Q(P)$. If a set of prices, P_0, is selected and the corresponding Q_0 is chosen as

$$Q_0 = Q(P_0)$$

then using the own price elasticities implied by Q, an approximate price function with zero cross-elasticities can be constructed such that

$$P_0 = P_d(Q_0). \tag{12}$$

This approximating price function is employed to produce problem (10).

If, in the solution of (10), the $Y_{i,k}$ increments satisfy

$$\sum_{k=1}^{n} (Y_{i,k} - Y_{i,-k}) = 0 \tag{13}$$

for all i, then by (11) it follows that

$$\pi \approx P_0 \tag{14}$$

and Q_0 is an approximate solution to (1) with the general demand function.

If (12) is not satisfied then by (11), π is an estimate of a new set of demand prices that would produce equilibrium. This is the heuristic step. The quality of this estimate should be related to the degree of dominance of the own price elasticities. In the approximating problem, the estimate is exact for the important special case of zero cross-elasticities. Furthermore, the iteration on these prices simulates a market decision process where individual product demand choices are made with only periodic information about the prices of all other products. This leads to the expectation of a successful iteration to a solution of (1) via the following algorithm.

(a) *Computational procedure.*

Step 1. Choose a set of demand prices, P^1. Let $t = 1$.

Step 2. Calculate $Q^t = Q(P^t)$. Using the own price elasticities of $Q(\cdot)$, construct (1) relative to the point (Q^t, P^t).

Step 3. Obtain (X^t, Y^t, π^t) as an optimal solution for (10). If $\pi^t = P^t$, go to Step 4. Else, let $P^{t+1} = \pi^t$, $t = t+1$ and go to Step 2.

Step 4. Terminate with equilibrium supply pattern X^t, consumptions Q^t and market prices P^t.

Formally, this interactive scheme is attempting to solve a fixed point problem. The convergence properties in the presence of a general demand function have not been established. The well-behaved convergence properties of related processes for general equilibrium models are discussed in length in [2]. Unfortunately, this problem does not satisfy the gross substitutability or homogeneity assumptions required by those results. Quadratic optimization and complementarity approaches are indicated in [12] which contains the most comprehensive discussion of the computational formulations and experience.

A1.4. *Construction of the supply model*

The preparation of the linear program describing the supply system (4) involves a large data collection and model formulation effort. The specification of the example in Appendix 2 illustrates the model for a simple energy system problem.

The supply model is conceptually equivalent to the PACE Energy Hydrocarbons System[6] or the static version of the Emergency Energy Capacity Model developed for the Office of Emergency Preparedness[5]. For simplicity, the basic structure is outlined here in terms of three major activities; production, conversion, and transportation.

Let

V_{ij} Production of product increment i in region j.
R_{jm} Operation of refining or conversion plant m in region j.
T_{ijkl} Transportation of product i from region j to region k via mode l.

†Note that convexity ensures $Y_{i,k}Y_{i,-k} = 0$. Further, if $Y_{i,k} > 0$ then $Y_{i,k-1} = U_{i,k-1}$.

‡A small but important change has occurred here. The π values for (10) are applicable to (4) but the converse may not be true. The problem in (10) differs from (4) in that it includes the opportunity cost of competing demands. This is the correct model and it compensates for continuity problems in (4). In a limiting sense, the equilibrium π values in (4) and (10) are equivalent.

The first constraints on the supply system establish bounds for each production increment limited by the value UV_{ij};

$$0 \le V_{ij} \le UV_{ij}.$$

bounds for transportation limited by value UT_{ijkl};

$$0 \le T_{ijkl} \le UT_{ijkl}$$

and bounds for conversion activities limited by value UR_{jm};

$$0 \le R_{jm} \le UR_{jm}.$$

For each producing region and each product it is necessary that total mass be preserved, hence the transportation out must be less than or equal to the total production;

$$-V_{ij} + \sum_{l} \sum_{k} T_{ijkl} \le 0.$$

Similarly, we need to preserve mass in each of the conversion or refining regions subject to the changes implied by the conversion technology. If r_{im} denotes the input ($r_{im} < 0$) or output ($r_{im} > 0$) of product i per unit of activity of plant m, this implies

$$\sum_{l} \sum_{k} T_{ijkl} - \sum_{l} \sum_{k} T_{ijkl} - \sum r_{im} R_{jm} \le 0.$$

In addition, there are joint limitations on transportation by any mode limited by the value BT_{jkl};

$$\sum_{i} T_{ijkl} \le BT_{jkl}.$$

Constraints on refining capacities are also required with upper limit BR_j;

$$\sum_{m} R_{jm} \le BR_j.$$

These constraints are supplemented by limitations on the availability of key resources (e.g., steel) or the maximum production of certain items (e.g., pollutants). If v_{ijn}, t_{ijkln}, and r_{jmn} represent the requirements for a resource n per unit of production, transportation or conversion, then the aggregate constraint on availability of the cross cut resource can be limited to value BC_n via the inequality,

$$\sum_{i} \sum_{j} v_{ijn} V_{ij} + \sum_{i} \sum_{j} \sum_{k} \sum_{l} t_{ijkln} T_{ijkl} + \sum_{j} \sum_{m} r_{jmn} R_{jn} \le BC_n.$$

This collection of constraints depicts the structure and intent of the set $\{X | X \in x, AX = b\}$ used in the development of the general algorithm.

These constraints must be combined with the demand equations represented as

$$\sum_{j} \sum_{l} T_{ijkl} = D_{ik}$$

where D_{ik} is the demand for the ith product in the kth region.

The complete outline of the supply linear program includes an objective function where the coefficients represent the minimum acceptable price for each unit of activity (v_{ijo}, t_{ijklo}, r_{jmo});

$$\text{Min} \sum_{i} \sum_{j} v_{ijo} V_{ij} + \sum_{i} \sum_{j} \sum_{k} \sum_{l} t_{ijklo} T_{ijkl} + \sum_{j} \sum_{n} r_{jmo} R_{jm}.$$

This linear program is used to generate the iterating problem in the algorithm for finding an equilibrium solution. At equilibrium, the dual values for the demand equalities equal the prices used to generate the demands D_{ik}. The interface used to describe these demand equations is characterized in the next section.

A1.5. *Construction of the demand model*

The demand model is based on a separate econometric system of behavioral equations which relate the future demand for energy products to prices and other economic or demographic variables. The details of this model are described in a separate section of the report.

The procedure for linking this demand model to the intergrating model is straightforward and permits the introduction of many variations in the demand component.

The dynamic demand model operates using a macroeconomic forecast to establish the basic environment for the energy demand projections. This macro forecast is combined with a large system of equations which estimate demand as a function of economic variables and prices. Holding the nonprice explanatory variables constant, this price and quantity forecast is interpreted as a point on the future demand curve. Subsequently, this price and quantity forecast is numerically perturbed for one price and differenced to obtain an estimate of the elasticities with respect to that price, again for each relevant year. Formally, the basic forecast produces an estimate of the demands Q^0_{it} and prices P^0_{it} for each product i and period t. The perturbed prices P^1_{it} consist of P^0_{it} for all but one i for which $P^1_{it} = (1 + \epsilon)P^0_{it}$. The perturbed quantities Q^1_{it} are the demands forecasted as a function of the perturbed prices. The elasticity of the demand for product i is a function of the price j, both in period t, is then estimated as

$$e_{ijt} = \frac{(Q^1_{it} - Q^0_{it})}{Q^1_{it} + Q^0_{it}} \cdot \frac{2}{\epsilon}.$$

This produces an array

$$E_t = (e_{ijt})$$

which is the table of elasticities and cross elasticities, changing over time, that are implied by the equations of the demand model.

Given the trajectory $(Q^0_{it}, P^0_{it}, E_t)$, the demand equation for each year is estimated as

$$\ln Q_{it} = \ln A_{it} + \sum_j e_{ijt} P_{jt}$$

where

$$\ln A_{it} = \ln Q^0_{it} - \sum_j e_{ijt} \ln P^0_{jt}.$$

This time dependent but constant elasticity approximation to the complex demand model is used to interface with the computational procedure as described in this Appendix.

The constant elasticity assumption becomes less tenable as the equilibrium solution moves further and further from Q^0 and P^0. The demand estimate is carefully constructed to approximate the final solution and improve the accuracy of the interface.

This approximation scheme permits the connection of the supply model with any demand model which can produce an estimate of a point on a demand curve and the relevant elasticities (Q_0, P^0, E). This flexibility has been exploited in the continued restructuring and improvement of the demand model and the use of the demand model to describe conservation policies or other questions which arise as shifts in the demand curves.

APPENDIX 2: SPECIFICATION OF EXAMPLE PROBLEM

The example problem of section 4 is designed to present the conceptual structure of the basic evaluation system. The computational translation of the verbal problem centers on the description of the linear program which approximates the energy supply and distribution system. The linear program for the example problem employs the following names for the decision variables in the network:

$C_{i,j}$ Coal in region i at increment j
$O_{i,j}$ Oil in region i at increment j
$CT_{i,k}$ Coal transport from region i to region k
$OT_{i,k}$ Oil transport from region i to region k
R_i Level of operation of refinery i
$L_{i,k}$ Light oil transported from region i to region k
$H_{i,k}$ Heavy oil transported from region i to region k
DL_k Demand for light in region k
DH_k Demand for heavy in region k
DC_k Demand for coal in region k
PL_k Price of light oil in region k
PH_k Price of heavy oil in region k
PC_k Price of coal in region k
S Steel availability
K Capital availability.

The basic network for the example is illustrated in Fig. A1. The constraints and relationships that describe this network are presented for each major category.

1. Coal production limits, region 1

$$O \le C_{1,1} \le 300$$
$$O \le C_{1,2} \le 300$$
$$O \le C_{1,3} \le 400$$

2. Coal production limits, region 2

$$O \le C_{2,1} \le 200$$
$$O \le C_{2,2} \le 300$$
$$O \le C_{2,3} \le 600$$

3. Oil production limits, region 1

$$O \le O_{1,1} \le 1100$$
$$O \le O_{1,2} \le 1200$$

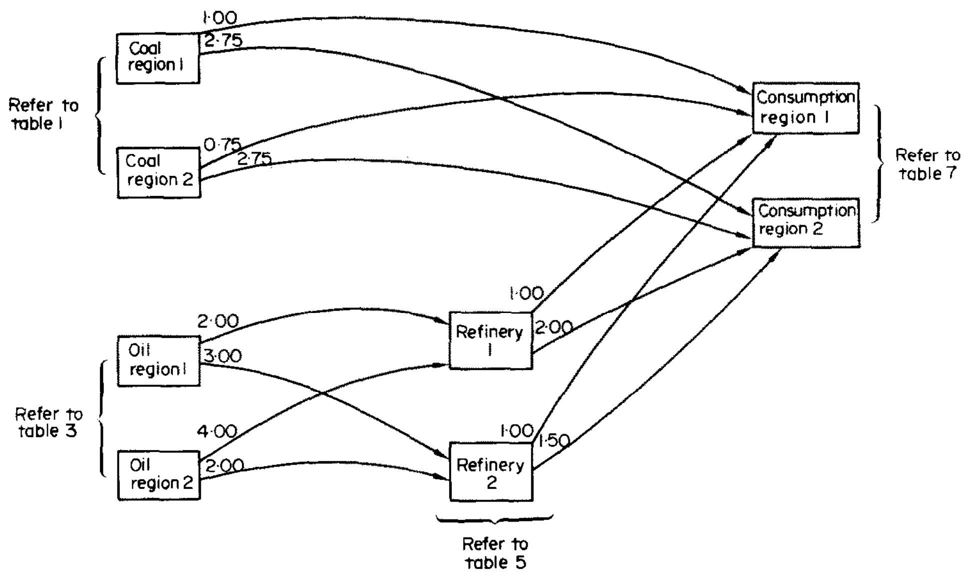

Fig. A1. Example energy system network.

4. Oil production limits, region 2

$$O \le O_{2,1} \le 1300$$

$$O \le O_{2,2} \le 1100$$

5. Mass balance, coal region 1

$$\sum_{j=1}^{3} C_{1,j} - \sum_{k=1}^{2} CT_{1,k} = 0$$

6. Mass balance, coal region 2

$$\sum_{j=1}^{3} C_{2,j} - \sum_{k=1}^{2} CT_{2,k} = 0$$

7. Mass balance, oil region 1

$$\sum_{j=1}^{2} O_{1,j} - \sum_{k=1}^{2} OT_{1,k} = 0$$

8. Mass balance, oil region 2

$$\sum_{j=1}^{2} O_{2,j} - \sum_{k=1}^{2} OT_{2,k} = 0$$

9. Crude mass balance, refinery region 1

$$\sum_{i=1}^{2} OT_{i,1} - R_1 = 0$$

10. Crude mass balance, refinery region 2

$$\sum_{i=1}^{2} OT_{i,2} - R_2 = 0$$

11. Light oil mass balance, refinery region 1

$$\sum_{k=1}^{2} L_{1,k} - 0{\cdot}6R_1 = 0$$

12. Light oil mass balance, refinery region 2

$$\sum_{k=1}^{2} L_{2,k} - 0{\cdot}5R_2 = 0$$

13. Heavy oil mass balance, refinery region 1

$$\sum_{k=1}^{2} H_{1,k} - 0\cdot 4R_1 = 0$$

14. Heavy oil mass balance, refinery region 2

$$\sum_{k=1}^{2} H_{2,k} - 0\cdot 5R_2 = 0$$

15. Coal demand balance, region 1

$$\sum_{i=1}^{2} CT_{i,1} = DC_1$$

16. Coal demand balance, region 2

$$\sum_{i=1}^{2} CT_{i,2} = DC_2$$

17. Light oil demand balance, region 1

$$\sum_{i=1}^{2} L_{i,1} = DL_1$$

18. Light oil demand balance, region 2

$$\sum_{i=1}^{2} L_{i,2} = DL_2$$

19. Heavy oil demand balance, region 1

$$\sum_{i=1}^{2} H_{i,1} = DH_1$$

Table A1. Equilibrium energy balances for example

Variable	Without Resource constraints	With Resource constraints
$C_{1,1}$	300	300
$C_{1,2}$	300	300
$C_{1,3}$	400	206
$C_{2,1}$	200	200
$C_{2,2}$	300	300
$C_{2,3}$	600	600
$CT_{1,1}$	2	0
$CT_{1,2}$	998	806
$CT_{2,1}$	1100	996
$CT_{2,2}$	0	104
$O_{1,1}$	1100	1100
$O_{1,2}$	1114	989
$O_{2,1}$	1300	1300
$O_{2,2}$	1100	1063
$OT_{1,1}$	2100	2089
$OT_{1,2}$	104	0
$OT_{2,1}$	0	0
$OT_{2,2}$	2400	2363
R_1	2110	2089
R_2	2504	2363
$L_{1,1}$	0	24
$L_{1,2}$	1226	1229
$L_{2,1}$	1252	1181
$L_{2,2}$	0	0
$H_{1,1}$	0	0
$H_{1,2}$	844	835
$H_{2,1}$	1041	996
$H_{2,2}$	211	185
PC_1	9·30	11·30
PC_2	11·00	13·40
PL_1	12·50	15·40
PL_2	12·60	15·60
PH_1	9·40	11·50
PH_2	9·40	12·00

20. Heavy oil demand balance, region 2

$$\sum_{i=2}^{2} H_{i,2} = DH_2$$

21. Joint steel constraint

$$1C_{1,1} + 2C_{1,2} + 3C_{1,3} + 1C_{2,1} + 4C_{2,2} + 5C_{2,3} + 4O_{1,2} + 2O_{2,2} \leq S$$

22. Joint capital constraint

$$1C_{1,1} + 5C_{1,2} + 10C_{1,3} + 1C_{2,1} + 5C_{2,2} + 6C_{2,3} + 10O_{1,2} + 15O_{2,2} \leq K$$

23. Supply price function

$$\begin{aligned}
&5{\cdot}00C_{1,1} + 6{\cdot}00C_{1,2} + 8{\cdot}00C_{1,3} + 4{\cdot}00C_{2,1} + 5{\cdot}00C_{2,2} + 7{\cdot}00C_{2,3} \\
&\quad + 1{\cdot}00O_{1,1} + 1{\cdot}50O_{1,2} + 1{\cdot}25O_{2,1} + 1{\cdot}50O_{2,2} \\
&\quad + 1{\cdot}00CT_{1,1} + 2{\cdot}50CT_{1,2} + 0{\cdot}75CT_{2,1} + 2{\cdot}75CT_{2,2} \\
&\quad + 2{\cdot}00OT_{1,1} + 3{\cdot}00OT_{1,2} + 4{\cdot}00OT_{2,1} + 2{\cdot}00OT_{2,2} \\
&\quad + 6{\cdot}50R_1 + 5{\cdot}00R_2 \\
&\quad + 1{\cdot}00L_{1,1} + 1{\cdot}20L_{1,2} + 1{\cdot}00L_{2,1} + 1{\cdot}50L_{2,2} \\
&\quad + 1{\cdot}00H_{1,1} + 1{\cdot}20H_{1,2} + 1{\cdot}00H_{2,1} + 1{\cdot}50H_{2,2}
\end{aligned}$$

For a fixed set of prices and demands, the supply problem reduces to the minimization of the price function (23), choosing among the decision variables subject to the various constraints (1)–(22).

Associated with each demand constraint (15)–(20), there is a supply price that is estimated by the solution of the linear program. This supply price will be equal to the demand price if the system is in equilibrium. If not, the procedure outlined in Appendix 1 is applied to estimate a new demand price, adjust the demands consistent with the elasticities, and prepare a new supply linear programming problem. This process is continued until an approximation to an equilibrium solution is obtained.

The example problem was solved twice in this manner. Section 4.1 discusses the case when no joint resource constraints are imposed. Section 4.2 presents the same problem with limitations on steel and capital availability. The detailed values of the decision variables for these two cases are presented in Table A2.1.