Calibrated Concave Preferences

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Jim has two girl friends, Nancy and Louise. A date with either girl places demands on Jim's two primary resources, time and money, both of which are exogenously given. Dates with Nancy may be brief but they are often expensive, typically involving jazz clubs or expensive restaurants. Louise likes bike rides and canoe trips, so dates with her are not so expensive but require more time.

Jim currently spends one hour a week with Nancy and two hours with Louise. The relative money price of an hour with Nancy is $\bar{p}$. Consistent with societal norms, we may presume that Jim's preferences are concave. In the absence of a time constraint, he would only date Louise. In the absence of a money constraint, he would only date Nancy.

His preferences are characterized by the following constant elasticity of substitution utility function:

$$U(x,y) = \phi\left(\alpha x^{\rho} + \beta y^{\rho}\right)^{1/\rho}$$

in which x represents hours spent with Nancy, y represents hours spent with Louse, and $\rho = 1 + 1/\eta$ where $\eta \geq 0$ is the elasticity of transformation. ϕ, α and β are exogenous constants which together with η characterize Jim's preferences. The value of ϕ is chosen such that $\bar{U} = U(x = 1, y = 2) = 1$.

i. Let θ be defined as:

$$\theta = \frac{\bar{\mu}\bar{x}}{\bar{\mu}\bar{x} + \bar{y}} = \frac{\bar{\mu}}{\bar{\mu} + 2}$$

where $\bar{\mu}$ is the benchmark *marginal rate of substitution*:

$$\bar{\mu} = \left.\frac{\partial U/\partial x}{\partial U/\partial y}\right|_{x=1,y=2}$$

Show U can be equivalently expressed as

$$U(x,y) = \left(\theta x^{\rho} + (1-\theta)(y/2)^{\rho}\right)^{1/\rho}$$

ii. Sketch Jim's budget set and benchmark indifference curve. Given existing time and money constraints, what are the maximum number of hours which Jim could spend with Louise? What are the maximum number of hours he could spend with Nancy?

iii. What values of θ are consistent with Jim's current choices if $\eta = +\infty$?

iv. What values of θ are consistent with Jim's current choices if $0 < \eta < \infty$?

v. What is the maximum level of concavity consistent with Jim's choices?