The basic closed-economy general-equilibrium model as an MCP

James R. Markusen University of Colorado, Boulder

Notes 2

(1) Multiple interacting agents

(2) Individual behavior based on optimization

(3) Most interactions are mediated by markets and prices

(4) Equilibrium: endogenous variables adjust such that

(a) agents, subject to the constraints they face, cannot do better by altering their behavior

(b) markets (generally, not always) clear so, for example, supply equals demand in each market.

Steps in Applied General-Equilibrium Modeling

(1) Specify dimensions of the model.

- Numbers of goods and factors
- Numbers of consumers
- Numbers of countries
- Numbers of active markets

(2) Chose functional forms for production, transformation, and utility functions; specification of side constraints.

- Includes choice of outputs and inputs for each activity
- Includes specification of initially slack activities

(3) Construct micro-consistent data set.

- Data satisfies zero profits for all activities, or if profits are positive, assignment of revenues
- Data satisfies market clearing for all markets

(4) Calibration – parameters are chosen such that functional forms and data are consistent.

- By “consistent”: data represent a solution to the model

(5) Replication – run model to see if it reproduces the input data.

(6) Counter-factual experiments.

Example: 2-good, 2-factor closed economy with fixed factor endowments, one representative consumer.

Simply economy, two sectors (X and Y), two factors (L and K), and one representative consumer (utility function W).

L and K are in inelastic (fixed) supply, but can move freely between sectors.

p_x, p_y, p_l, and p_k are the prices of X, Y, L and K, respectively.

I is consumer's income and p_w will be used to denote the price of one unit of W. p_w is thus the consumer price index.

(1) $X = X(L_x, K_x)$

(2) $Y = Y(L_y, K_y)$

(3) $L^* = L_x + L_y$

(4) $K^* = K_x + K_y$

(5) $W = W(X, Y)$

(6) $CONS = p_l L^* + p_k K^* = p_x X + p_y Y$

How do we find equilibrium: prices, and factor allocations?

Equilibrium could be solved for by a constrained optimization problem: Max (5) subject to the constraints (1), (2), (3), (4), (6).

The usefulness of this approach breaks down quickly as the model becomes more complicated.

Alternative approach: convert the problem to a system of equations, and solve that system.

Solve the cost minimization problems for producers and consumers: individual optimizing behavior is embedded in the model.

These give the minimum cost of producing a good.

Unit cost functions for X and Y $cx = cx(p_l, p_k)$,
$cy = cy(p_l, p_k)$

Unit cost (expenditure) function for W $e = e(p_x, p_y)$

The next crucial step is provided by theory: Shepard's lemma

$$\frac{\partial cx}{\partial p_l} = cx_{pl} =$$ X demand for labor per unit of output

$$\frac{\partial cx}{\partial p_k} = cx_{pk} =$$ X demand for capital per unit of output

$$\frac{\partial e}{\partial p_x} = e_{px} =$$ Consumer's demand for X per unit of utility

General equilibrium is the solution to a square system of 9 weak inequalities in 9 unknowns.

These 9 fall into three catagories, and this is a very general structure for applied general-equilibrium models.

(A) pricing inequalities (zero-profit inequalities)
(B) market clearing inequalities
(C) income balance equations

These weak inequalities can be solved for the unknowns X, Y, W, p_x, p_y, p_w, p_l, p_k, and I.

Formulating equilibrium as a complementarity problem requires that each inequality is associated with a particular variable.

If a zero profit conditions holds as a strict inequality in equilibrium, profits for that activity are negative, that *activity* will not used.

The complementary variable to a *zero-profit condition* is a *quantity*, the activity level.

If a market-clearing condition holds as a strict inequality, supply exceeds demand for that good or factor in equilibrium so its price must be zero.

The complementary variable to a *market clearing inequality* is the *price* of that good or factor.

The complementary variable to an income balance equation is just the income of that agent.

(1) Non-positive profits for X	$cx(p_l, p_k) \geq p_x$	X
(2) Non-positive profits for Y	$cy(p_l, p_k) \geq p_y$	Y
(3) Non-positive "profits" for W	$e(p_x, p_y) \geq p_w$	W
(4) Supply $\geq$ Demand for X	$X \geq e_{px}(p_x, p_y)W$	p_x ,
(5) Supply $\geq$ Demand for Y	$Y \geq e_{py}(p_x, p_y)W$	p_y
(6) Supply $\geq$ Demand for W	$W \geq I / p_w$	p_w
(7) Supply $\geq$ Demand for L	$L^* \geq cx_{pl}X + cy_{pl}Y$	p_l
(8) Supply $\geq$ Demand for K	$K^* \geq cx_{pk}X + cy_{pk}Y$	p_k
(9) Income balance	$I = p_l L^* + p_k K^*$	I

2 Micro consistent data

A data set is micro consistent when it satisfies the conditions for economic equilibrium (it could be generated as the solution to some model).

Data must satisfy *zero profits*, *market clearing*, and *income balance*.

The above problems can be thought of as consisting of
three production activities, X, Y, and W,
five markets, X, Y, L, K and W
income balance

Represent the initial data for this economy by a rectangular matrix. We could call this a micro-consistent data matrix.

C*olumns* corresponding to *production activities(sectors)* and *consumers*.

Rows correspond to *markets*. Complementary variables are prices, so we have listed the price variables on the left to designate rows.

A positive entry signifies a receipt (sale) in a particular market. A negative entry signifies an expenditure (purchase) in a market.

The numbers of the matrix are *values, prices times quantities*. The modeler decides how to interpret these as prices or quantities.

The data is balanced or “micro-consistent” when row and column sums are zeros.

	Production Sectors			Consumers	
Markets	X	Y	W	CONS	Row sum
PX	100	100	-100		0
PY		100	-100		0
PW			200	-200	0
PL	-25	-75		100	0
PK	-75	-25		100	0
Column sum	0	0	0	0	

Row sum = 0 = *market clearance*, and one such condition applies for each commodity in the model.

Column sums = 0 = zero profits or “product exhaustion” in an alternative terminology.

3 Calibration and replication

Calibration is choosing functional forms and their parameters such that the initial micro-consistent data is a solution to the model.

We use Cobb-Douglas functions for the three activities. The share parameters for the functions are given in the data matrix above.

Goods in the utility function get equal shares of 0.5:

$$e(p_x, p_y) = p_x^{0.5} p_y^{0.5}$$

X is capital intensive: a capital share of 0.75, a labor share of 0.25.

$$c_x(p_l, p_k) = p_l^{0.25} p_k^{0.75}$$

Y is labor intensive with the opposite ordering of shares.

$$c_y(p_l, p_k) \;=\; p_l^{0.75} p_k^{0.25}$$

Example of Shepard's lemma: demand for labor in Y.

$$L_y \;=\; \frac{\partial c_y(p_l, p_k)}{\partial p_l} Y \;=\; 0.75 p_l^{0.75-1} p_k^{0.25} Y \;=\; 0.75 p_l^{0.75} p_k^{0.25} Y / p_l$$

If p_x is the producer price of X, then this can be written as:

$$L_y \;=\; \frac{\partial c_y(p_l, p_k)}{\partial p_l} Y \;=\; 0.75 p_x / p_l * Y$$

Replication check: run to model, should return initial data

This modeling framework can be extend to cover:

- taxes, subsidies, quotas, licenses, equal-yield tax reform
- multiple household types, setting endogenous taxes for income redistribution targets
- public consumption or intermediate goods, pollution, production externalities
- increasing returns and imperfect competition (monopolistic competition)
- increasing returns and imperfect competition (oligopoly - with or without free entry)
- open economy models

Turn now to GAMS models using the above formulation of 2-goods, 2-factors, one representative consumer in a closed economy.

madison1.gms	basic model from above, also from JGEA paper
madison2.gms	extension: tax versus trade/transactions cost Also in JGEA paper
madison3.gms	extension: quantitative restriction versus tax Also in JGEA paper
madison4.gms	extension: small open economy

Kyiv1.gms

Table 1: Closed economy results (variables normalized to equal 1 in benchmark)

Variable	Benchmark	Double labor Endowment	Double capital Endowment	Double both labor and capital
W	1.00	1.41	1.41	2.00
X	1.00	1.19	1.68	2.00
Y	1.00	1.68	1.19	2.00
PX	1.00	1.19	0.84	1.00
PY	1.00	0.84	1.19	1.00
PL	1.00	0.71	1.41	1.00
PK	1.00	1.41	0.71	1.00
PW	1.00	1.00	1.00	1.00
I	1.00	1.41	1.41	2.00

Notes to Table 1:

Column 2 illustrates aggregate diminishing returns

Column 3 illustrates the symmetry in the model

Column 4 illustrates homogeneity of the economy under constant returns in production and homothetic preferences

Kyiv1.gms

Table 2: Small open economy results

Variable	Benchmark	Double labor Endowment	Increase price of good X by 50%	Increase price of good X by 100%
W	1.00	1.50	1.08	1.24
X	1.00	0.50	1.56	1.75
Y	1.00	2.50	0.31	
PX	1.00	1.00	1.50	2.00
PY	1.00	1.00	1.00	1.00
PL	1.00	1.00	0.82	0.88
PK	1.00	1.00	1.84	2.63
PW	1.00	1.00	1.23	1.41
I	1.00	1.50	1.33	1.75

Notes to table 2: Column 2 illustrates the Rybczynski Theorem
Column 3 illustrates the Stolper-Samuelson Theorem
Column 4 illustrates the importance of not ruling out corner solutions (specialization)

Figure 1: Comparison of tax and transactions cost

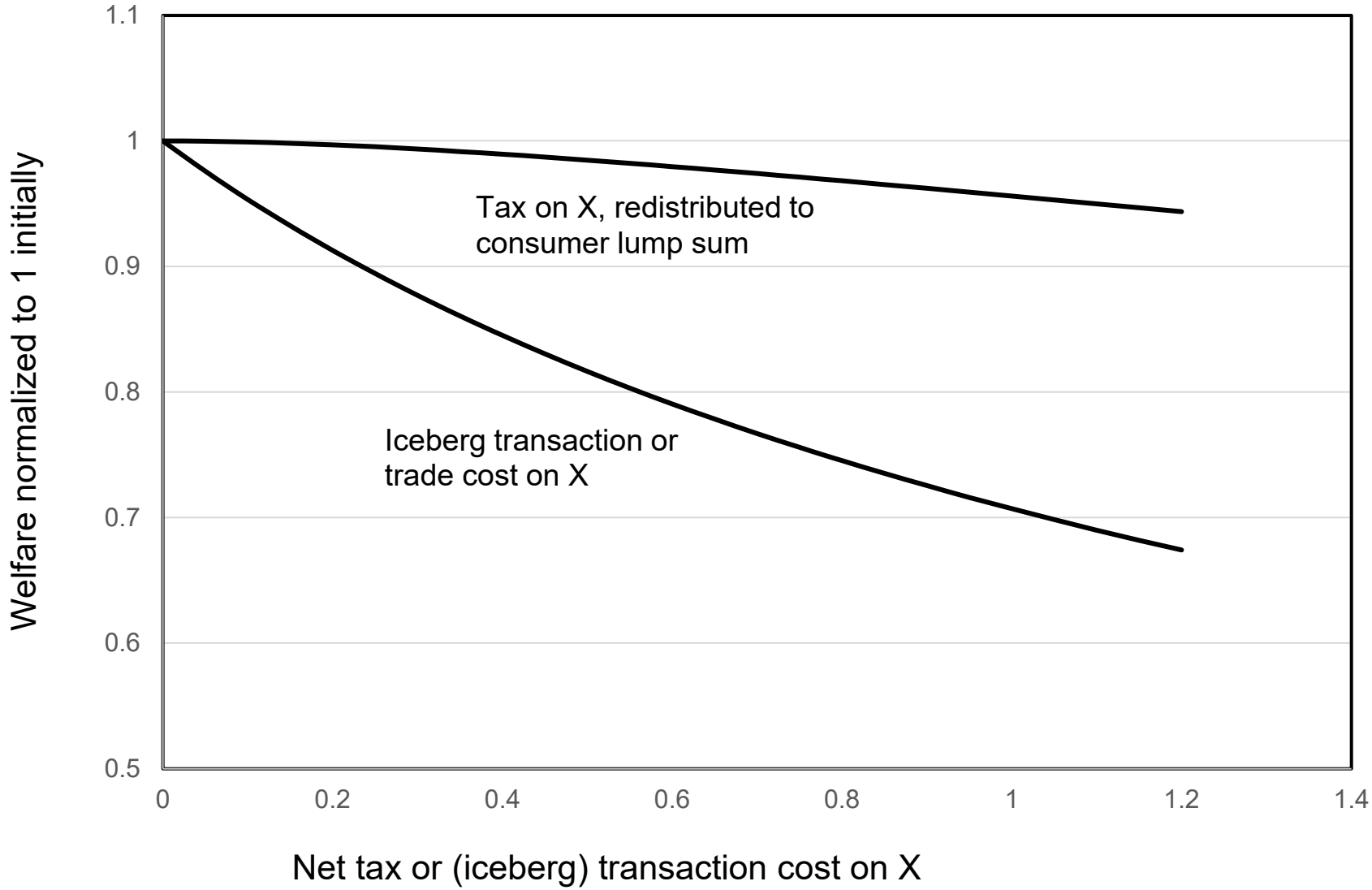
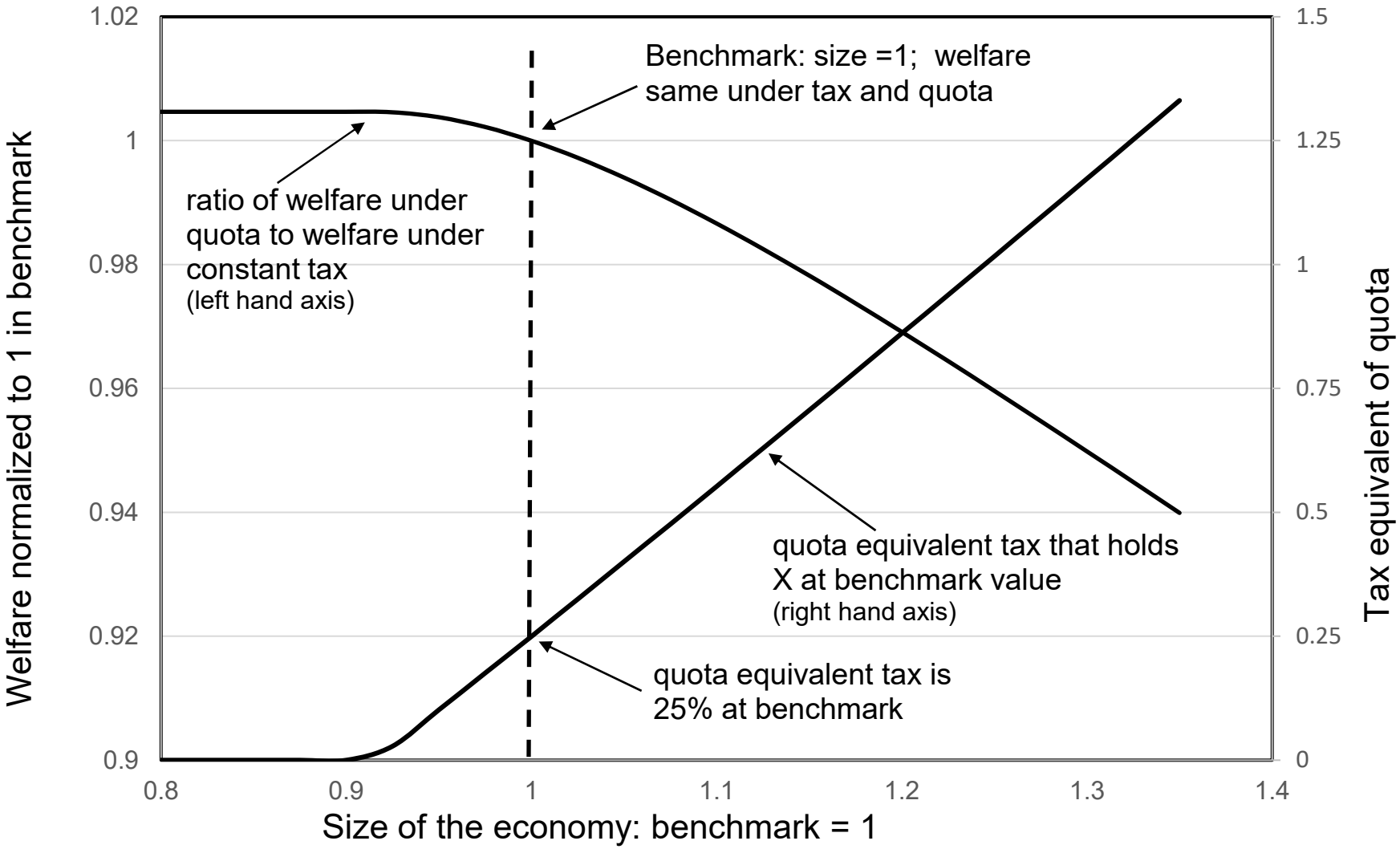


Figure 2: Non-equivalence of tax and quota as economy grows



Small open economy: madison4.gms

Small open economy 2x2 (two goods, two factors) in which the rest of the world is not explicitly modeled.

Know as the “small-country assumption”: the country faces fixed world prices

Trading opportunities are summarized by simple functions which allow the economy to transform one good (an export) into another (an import).

The “technology” of these functions represents world prices. We will assume that these technologies or price ratios are fixed in this model.

Country exports X_1 in exchange for X_2 (activities E1, M2)

We will allow for the fact that some policy or endowment change could actually reverse the direction of trade by specifying (initially inactive) functions that transform goods in the opposite direction.

Initial data, in which 50 units of good X_1 are exchanged for 50 units of good X_2 at an implicit price ratio of $p_1/p_2 = 1$.

	Production Sectors					Consumer
Markets	X1	X2	E1	M2	W	CONS
P1	150		-50		-100	
P2		50		50	-100	
PL	-135	-5				140
PK	-15	-45				60
PW					200	-200
PFX			50	-50		

Technology parameters are specified in these functions that allow the modeler to change the terms of trade.

```
PARAMETERS
   PE2      Export price of good 2,
   PM1      Import price of good 1,
   PE1      Export price of good 1,
   PM2      Import price of good 2,
   TM2      Import tariff for good 2;

PE1 = 1;
PM2 = 1;
PE2 = 0.999;
PM1 = 1.001;
```

E1 and M2 production activities that are the initially active trade links.

Small trade costs on inactive links prevent model degeneracy and "round tripping".

If all prices = 1, then if E1 = M2 = 1 is a solution, then so is E1 = 2, M1 = 1, M2 = 1.

While we could specify this activity as directly transforming X_1 into X_2, in more complicated models it proves useful to define another good which we will call “foreign exchange” and whose price is denoted PFX.

All trade is mediated through the “foreign exchange market”.

Thus activity E1 transforms X_1 into foreign exchange and M2, the import activity for good 2, transforms foreign exchange into imports of good 2.

There are four trade activities as just noted.

```
PRF_E1..  0*P1 =G= 50*PFX*PE1;

PRF_E2..  50*P2 =G= 50*PFX*PE2;

PRF_M1..  50*PFX*PM1 =G= 50*P1;

PRF_M2..  50*PFX*PM2 =G= 50*P2;
```

The equation for “foreign exchange is the trade balance condition:

```
MKT_PFX.. 50*E2*PE2 + 50*E1*PE1 =G= 50*PM2*M2 + 50*PM1*M1;
```